universal calculation of piece values

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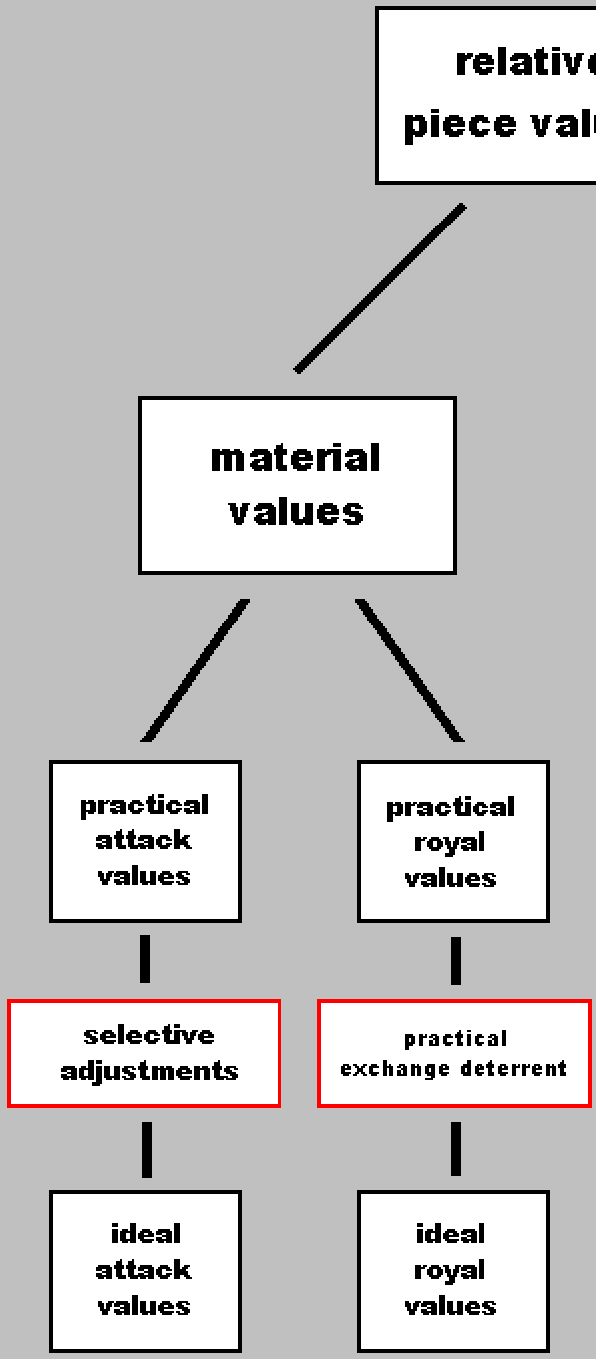
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introduction

A universal method for the estimation of "material values" applicable to chess variants (by the restrictive, proper definition), regardless of the pieces and board geometries involved, is achieved herein although some radical designs cause problems. The improvement of this model is an ongoing effort.

There are three main, legitimate types of models for calculating-estimating the material values of pieces upon boards: constant-value, variable-value unilateral and variable-value bilateral.

The constant-value model is the simplest yet intransigently, grossly inaccurate due to the irrefutable fact that the relative, material values of pieces change significantly during the course of any game. An example of it is the familiar list of the material values of pieces in chess. In fact, the material values of pieces in chess vary between the opening game, mid-game and endgame. So, it is merely a crude, easily-memorable guideline.

The variable-value unilateral model is of moderate complexity yet limited in accuracy due to the irrefutable fact that the relative, material values of one players' pieces are affected by the comparative strength and distribution of pieces belonging to one's opponent IF unequal. Of course, it is unlikely that both players will comparatively have exactly the same pieces after the late opening game when a few captures probably will have occurred. The most serious shortcoming for this model resides with the critically-important estimation of the value of a player's royal piece (which depends upon the material value of the opponent's army minus the royal piece).

The variable-value bilateral model is the most complex yet potentially accurate without any fundamental, theoretical limitation. This is the type of model presented herein.

Note that although only calculations involving square-spaced boards are provided, calculations for triangle-spaced or hexagon-spaced boards could easily be adapted for and accommodated by changing the assumptions involving available, geometrically-contiguous directions of movement.

On square-spaced boards, there are a maximum of 8 geometrically-contiguous directions of movement available.

On triangle-spaced boards, there are a maximum of 12 geometrically-contiguous directions of movement available.

On hexagon-spaced boards, there are a maximum of 6 geometrically-contiguous directions of movement available.

A methodical build-up of foundations is followed from the most basic, useful calculation method (i.e., "ideal attack values") as a starting point to the highest achieved estimation method (i.e., "material values") as a finishing point … with only a supplemental, general explanation of the desired yet out-of-reach, ultimate goal (i.e., "relative piece values").

The journey is typically much more successful and mathematically detailed at the start than at the finish with calculations involving constants used at the beginning and middle but estimates involving constants used at the end. Where estimates involving variables would have begun, the project ends.

Everything possible has been done to establish an understandable, holistic reference of the various terms, concepts and factors in relation to one another and moreover, give a vivid hierarchal image of the various levels at which all relevant inputs operate as methods, measurements, calculations or estimates.

The calculation method used herein is original and unique overall from any I know of that is published and/or used as an algorithm within any chess variant program.

The only foundation for this calculation method is "ideal attack values" which are refined in an entirely original manner in overlying layers.

As the diagram indicates, there are four levels of methods, measurements, calculations and/or estimates that must all be solved to have any chance of eventually achieving complete, reasonably-accurate, "relative piece values".

[Note- The various sub-calculations for some of the starting terms in the diagram are not shown.]

At the first level, there is really only the "ideal attack values" calculation since it is pointless to perform the "ideal royal values" estimate in isolation from and without reference to the other result (which is not possible to accomplish). The means to achieve this result is provided within this work.

At the second level, there is really only the "practical attack values" calculation since it is pointless to perform the "practical royal values" estimate in isolation from and without reference to the other result (which is not possible to accomplish). The means to achieve this result is provided within this work.

At the third level, there is the "material values" estimate where the "practical attack values" calculation and the "practical royal values" estimate are added together so that complete values are attained for all royal pieces. The means to achieve this result is provided within this work.

At the fourth level, there is the "relative piece values" estimate where the "material values" estimate and the "positional values" estimate are multiplied together.

The complex and game-specific estimation of positional values is not attempted within this work or even possible since the exact position of every piece within a game in progress is prerequisite. In practice, "positional values" for pieces that are variables do not exist outside a working computer chess program with a game in progress. This is in sharp contrast to the situation with "material values" for pieces that are constants able to be represented in a neat table without reference to the positions of the pieces or the game state. Thus, the means to achieve this result is NOT provided within this work. Only a brief, general description of what positional values entail is given.

Ultimately, only three out of the four levels desired can be solved following the information provided within this work as a guideline. "Material values" can be estimated but true "relative piece values" are not approached at all by any method.

It is really not my intention to give chess variant players advice but I think it is important to issue a few warnings about how the results obtain thru this estimation method for material values should NOT be used.

I think it can safely be asserted that great players understand and appreciate that the levels of depth and irony within the tactics and strategy of chess variants go to such extremes that every rule has an exception and every exception has an exception.

Accordingly, material values should not be applied literally and to the extreme. They should only be used as tactical [not strategic] guidelines with rational limitations as an important consideration toward deciding in most (but not all) cases which exchanges to force or avoid. Furthermore, they should be used with caution, primarily referred to during the opening game (IF not following the preferred course- an opening book) and secondarily referred to during the midgame.

Wherever game-winning objectives are attainable thru material sacrifices (esp. during the endgame), positional play must be used exclusively (for survival or victory) instead of material values. For example, whenever "checkmate" is attainable regardless of material sacrifices, the concept of material values becomes naive and non-applicable.

An all-or-nothing expectation from any piece valuation method is unrealistic esp. when examples are posed that push game conditions to the extremes where applying material values without reservation is ill-advised. Besides, how often can one realistically expect an exchange 10 or so pieces deep to occur via forced lines of play? If you are a player involved in such a dangerous escalation, then you had better consider game-winning conditions (i.e., "positional values") as carefully as material, exchange values.

Of course, it is always extremely important to not mistakenly interchange the concepts of material values and relative piece values despite how common this mistake is amongst published materials.

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attack values, ideal and practical

Ideal attack values are constants determined by calculation.
Practical attack values are constants determined by calculation.

Ideal attack values are not game-specific. They are board-specific and many games can be played upon the same board.

Practical attack values are game-specific and <u>never</u> board specific.

Ideal attack values are calculated for pieces moving on an ideal, otherwise-empty gameboard. They are typically values that are too high since no realistic downward adjustment for obstacles is admitted within their calculation.

Practical attack values are calculated for pieces moving on a real, partially-occupied gameboard during the average point of the first 2/3 of a given game in progress where the pieces for white are concentrated on the south side and the pieces for black are concentrated on the north side.

Practical attack values are ideal attack values accompanied by the six needed adjustments (i.e., "selective adjustments") used within their calculation that ultimately have measurable bearing upon their material values and in turn, their relative piece values. They are typically values that are within the proper range.

The first three out of the six selective adjustments apply only to pieces of unlimited range (i.e., sliders) based upon game-specific measurements involving:

1. **8-directional measurements**

   The average extent of directional movement available in all 8 directions (for square spaced boards) for sliders within the board.

2. **average piece densities**

   The ratio between baseline average piece density (0.5) and the average piece density for the first 2/3 of a game.

3. **3-directional foci**

   The measured presence or absence of the capability to move in the 3 vital directions of attack for sliders (weighted by the relative importance of vertical and diagonal directions of attack).

The last three out of the six selective adjustments apply both to pieces of unlimited and/or limited range based upon game-specific measurements involving:

4. **non color-bound enhancement**

   The piece(s) in the game that possess a bishop component as well as an additional movement required to re-position and allow them to hit spaces of the opposite color upon their next diagonal bishop move are given an appropriate bonus <u>per piece</u> to its unadjusted practical attack value.

5. **non color-changed enhancement**

   The piece(s) in the game that possess a knight component as well as an additional movement required to re-position and allow them to hit spaces of the same color upon their next color-changing knight move are given an appropriate bonus <u>per piece</u> to its unadjusted practical attack value.

6. **compound enhancement**

   The piece(s) in the game that possess as components the entire movement capabilities of two or more other power pieces (such as a knight, bishop or rook) are given an appropriate bonus <u>per piece</u> to its unadjusted practical attack value.

Note- The first two selective adjustments (8-directional measurements & average piece densities) determine the selective move blocks. Later, when the selective move blocks and 3-directional foci are applied to ideal attack values, unadjusted practical attack values are calculated. Finally, when the unadjusted practical attack values are treated with the sum of the non color-bound enhancement, non color-changed enhancement and compound enhancement, practical attack values are calculated.

The three selective adjustments for sliders do not apply at all to pieces of limited (1-space or 1-leap) range.

They are multiplicative factors that apply only to pieces of unlimited range (i.e., sliders) since:

1. **With regard to 8-directional measurements …**

   A. **The likelihood that the movement of a piece from its origin will be blocked short somewhere along its path increases proportionally with the extent of its movement and the number of pieces upon the board.**

   B. **If blocked short, the fraction of its extent along its path that will go unused increases proportionally with the extent of its movement.**

2. **With regard to average piece densities …**

   Only pieces of unlimited range (i.e., sliders) have their potential movements hindered by crowding of the board with pieces.

3. **With regard to 3-directional foci …**

   Only pieces of unlimited range (i.e., sliders) can attack across the board (between the south army of white and the north army of black) in one move via the N-S vertical, NE-SW diagonal or NW-SE diagonal.

By inference, the three reasons listed above also explain why the three selective adjustments for sliders do <u>not</u> apply to pieces of limited range.

The practical attack values equal the ideal attack values multiplied by the factors representing all six selective adjustments.

Complete formulae for reliably calculating selective move blocks in terms of all efficacious factors are provided. A choice is available between a compound, universal formula for use where a mixture of pieces of unlimited and limited range exists or a simple, special formula for use where pieces of unlimited range (i.e., sliders) exclusively exist.

Practical attack values must be calculated and used instead of ideal attack values in all games with every imaginable set of pieces. In turn, this entails calculating at least five out of the six selective adjustments and applying them to all (or nearly all) pieces individually as necessary.

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description of ideal attack values calculation

The immediate purpose is to mathematically-geometrically calculate the ideal attack values (aka- “spaces-affected criterion”) component of the material values that can be handled quickly, simply, without value judgments and by a method universally-applicable to virtually all chess variants (by their restrictive, proper definition), regardless of their board geometries and pieces involved.

Ideal attack values can be manually calculated using a bit of visual geometry and arithmetic. It is unavoidably, moderately tedious and time-consuming yet a valuable, convenient, easily-remembered reference for human players. Ultimately, this calculation carries the advantage of being achievable within a reasonable time using only a few-several board diagrams, a calculator, pencil & paper.

how to calculate ideal attack values
(on square, triangle or hexagon spaced boards)

1. Print-out a diagram of an empty board.

2. Make copies, one per unique piece used within a given game.

3. For each unique piece, visually count how many pieces on spaces it can attack from every space upon the board it can occupy, writing this number into every space as you go. [Note- Never count the space it rests upon.]

   This is the “spaces-affected” criterion.

4. Add-up all of the numbers written into every space for a diagram dedicated to each unique piece. This determines the total strength for each unique piece upon the board (based upon the total spaces pieces can attack upon a given board).

5. Repeat until the set of total strengths of all of the pieces used within a given game have been calculated.

6. Compare their values. Find the neatest available empirical ratio for all pieces (to an accuracy of 1/100 of a point) where the least-valued piece equals exactly 10 points. This is the table of ideal attack values.

Shortcut-

Wherever there exist composite pieces, their ideal attack values do not have to be calculated manually since they can be accurately obtained simply by adding together the values of the component pieces (already calculated manually) that they move as.

4 directions of movement
(in opposite pairs)

[8-directional measurements]
[selective move blocks]
[selective adjustments]

Even when a piece of unlimited range (i.e., slider) is not blocked by any other piece in its path, the hard limiting factor to its movement of failsafe, default effect is the limits or edges of the gameboard itself. This must be taken into account as a downward adjustment. Consequently, the average number of square-spaces which can be moved to across an ideal, otherwise-empty gameboard needs to be counted for all eight possible directions of movement and applied appropriately to pieces via the directions they actually move. These are termed "directional averages".

Unless a game arbitrarily forbids using opposite directions of movement upon its board (which is very rare), this will reduce the number of unique, directional averages further to four. Still, a series of calculations involving a game's board and pieces are required to accomplish this without admitting inaccuracies thru the indiscriminant use of averages.

The average extent of movement across the board, measured in the number of square-spaces for four directions of movement (NE-SW diagonal, NW-SE diagonal, N-S vertical, E-W horizontal) in opposite pairs by every path must be determined. These four directional measurements are identical to the ideal attack values for pieces upon the given board with the described capabilities of movement. Specifically, the four 2-directional sliders- diagon I, diagon II, horizon, verizon. Even if these pieces are not used within a given game, calculate their ideal attack values upon its gameboard in preparation for their usage.

There are a maximum of 15 unique, possible combinations of directions of movement (in opposite pairs) available upon a board with a total of 2, 4, 6 or 8 directions of movement. However, only the four directional combinations consisting of two directions of movement are basic. All other 11 directional combinations consisting of 4, 6 or 8 directions of movement are composite, built upon 2, 3 or 4 of the 4 basic directional combinations. Consequently, original computation never requires more than the ideal attack values of the 2-directional sliders exhibiting the 4 basic directional combinations.

The movement capabilities of the pieces at hand determine which (if any) ideal attack values of pieces representing the four basic directional combinations need to be added together and averaged in an intermediate step toward determining practical attack values where pieces representing any-all of the 11 composite directional combinations are involved.

15 directional combinations

2 directions basic 1 component (directional average)	4 directions composite 2 components	6 directions composite 3 components	8 directions composite 4 components
NE-SW	N-S E-W	N-S E-W NE-SW	NE-SW NW-SE N-S E-W
NW-SE	NE-SW NW-SE	N-S E-W NW-SE	
N-S	N-S NE-SW	NE-SW NW-SE N-S	
E-W	N-S NW-SE	NE-SW NW-SE E-W	
	E-W NE-SW		
	E-W NW-SE		

average piece densities
[selective move blocks]
[selective adjustments]

The piece density is a relation between the board and its pieces defined at any given moment as the number of pieces on the board divided by its number of spaces.

piece density = pieces + spaces

A set of two average piece densities, baseline and first 2/3 of a game, need to be referenced within the calculation of selective move blocks IF a game involves a mixture of unlimited and limited range pieces. Most chess variants do. However, IF a game involves pieces of unlimited range (i.e., sliders) exclusively, then there is no need to calculate these terms since they do not alter the relative ratios internally between any pieces within this class. Average piece densities only alter the relative ratios externally between pieces of unlimited and limited range as entire classes.

The baseline average piece density is a non game-specific calculation.
It is universal. The baseline average piece density always equals exactly 0.5 for any given gameboard.

For an imaginary game, it is the average between the starting piece density where the board is completely full and the finishing piece density where the board is completely empty.

s = starting piece density = 1.0

f = finishing piece density = 0.0

b = baseline average piece density = (1.0 + 0.0) + 2 = 0.5

The first 2/3 (of a game) average piece density is a game-specific calculation. Only basic knowledge of a given game is needed to provide it.

For a real given game, it is the average between the starting piece density (defined by the rules of the game) and the piece density where exactly 1/3 of the starting piece density remains.

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s = starting piece density = $3t$
(defined)

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t = 1/3 piece density = $0.\overline{3}s$

---

m = first 2/3 average piece density = $(s + 0.\overline{3}s) + 2 = 0.\overline{6}s$

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description of 4 selective move blocks calculation

Either a computer with a scientific calculator program or a physical, scientific calculator is needed for the step where one must extract exponential roots of real numbers with accuracy.

how to calculate 4 selective move blocks
(on square-spaced boards only)

01. **For each ideal attack value of each piece representing one of the four basic directional combinations, convert into its cube roots.**

02. **Its absolute value of its cube root(s) must be taken, leaving only its positive cube root.**

03. **Its positive cube root must be converted into its reciprocal (multiplicative inverse).**

04. **Its reciprocal (multiplicative inverse) must be subtracted from +1 to obtain its difference.**

05. **Repeat until the set of all four differences is obtained. Just hold them.**

IF pieces of unlimited range (i.e., sliders) are exclusively used in the game, then skip the rest of the steps. Use the existing set of four differences as the set of four selective move blocks without reservation.

06. **Research the given game to calculate its first 2/3 (of a game) average piece density.**

07. **Divide the baseline average piece density (0.5) by its first 2/3 (of a game) average piece density to obtain its quotient.**

08. **Multiply the set of four differences (from step #5) by the quotient (from step #7) to obtain the set of four selective move blocks.**

selective move blocks
formulae

If a mathematically-competent person prefers not to use either of the above 5-step or 8-step procedures, then a choice between appropriate formulae that can be directly used is available:

x = ideal attack value (basic directional combination piece)
m = first 2/3 (of a game) average piece density
b = baseline average piece density = +0.5

mixed pieces (unlimited and limited range pieces)
universal formula

$$\text{selective move blocks} = \left(+1 - |3\sqrt{x}|^{-1}\right) \;\mathrm{X}\; (+0.5/m)$$

unlimited range pieces only
special formula

$$\text{selective move blocks} = +1 - |3\sqrt{x}|^{-1}$$

The first term of the universal formula involving 8-directional measurements is used to appropriately decrease the values for unlimited-range pieces relative to limited-range pieces based upon the limits the board places upon unlimited-range pieces for each defined direction of movement. It is a game-specific calculation that references the gameboard.

The second term of the universal formula involving average piece densities is used to appropriately adjust (decrease or increase) the values for unlimited-range pieces relative to limited-range pieces based upon how much more or less the first 2/3 (of a game) average piece density deviates from the baseline average piece density at 0.5.

In other words …

If the board is crowded (greater than 0.5 piece density) during the first 2/3 of a game, then the values for unlimited-range pieces will be decreased by appropriate measure.

If the board is spacious (lesser than 0.5 piece density) during the first 2/3 of a game, then the values for unlimited-range pieces will be increased by appropriate measure.

If the board is neither crowded nor spacious (exactly equal to 0.5 piece density) during the first 2/3 of a game, then the values for unlimited-range pieces will be left unchanged.

It is a game-specific calculation that references the gameboard, pieces and rules.

It is noteworthy that the first 2/3 of a game average piece density (which is vital to calculating practical attack values and beyond) concentrates upon the first 2/3 of a game (defined as where the first 2/3 of the pieces are captured) and totally neglects the last 1/3 of a game (defined as where the last 1/3 of the pieces are captured). This is not an accidental, hazardous omission. It is an intentional, prudent emphasis, instead.

Although all moves within a game are potentially critically important, it is well-established that relatively, all things otherwise equal, the most important move in the game is the very first and the least important move in the game is the very last. Accordingly, the opening game is generally more important than the midgame that is generally more important than the endgame. Game-winning advantages are often (as well as most effectively) established early in the game thru superior play.

In the endgame, even the best material values often become an unreliable or erratic basis for decisions esp. where game-winning objectives and positional play toward those objectives become theoretically possible, paramount concerns that render material values meaningless by comparison. Consequently, no attempt should be made to diplomatically calibrate, average and compromise material values as to be equally useful throughout all phases of the game. Although material values should be somewhat useful throughout nearly all of the game as a general guideline, their accuracy is primarily important during the opening game and secondarily important during the midgame. Finally, the use of material values at all during some phases of the endgame should be tentative since it is of uncertain merit.

Accordingly, practical attack values, the main foundation for estimating material values, are optimized to be most accurate and useful throughout the opening game and the midgame (i.e., the first 2/3 of a game).

3-directional foci
[selective adjustments]

The distinction between indiscriminant, often-ineffective attack capabilities in all eight directions and discriminant, often-effective attack capabilities in the three vital directions is made at this juncture.

Since the vast majority of 2-player chess variants have opening setups in which the army of the white player is concentrated upon the south side of the board and the army of the black player is concentrated upon the north side of the board, effective attacks between the two armies (esp. in the opening game) usually entail moves beyond limited range along the N-S vertical, NE-SW diagonal or NW-SE diagonal directions.

Specifically …

For the white player, the most likely effective directions of attack are N, NE & NW.
For the black player, the most likely effective directions of attack are S, SE & SW.

Furthermore, the one vertical direction of attack (N or S) is more likely to be an effective direction of attack than either of the two diagonal directions of attack (NE or SW, NW or SE). Hence, it is more highly valued within calculations.

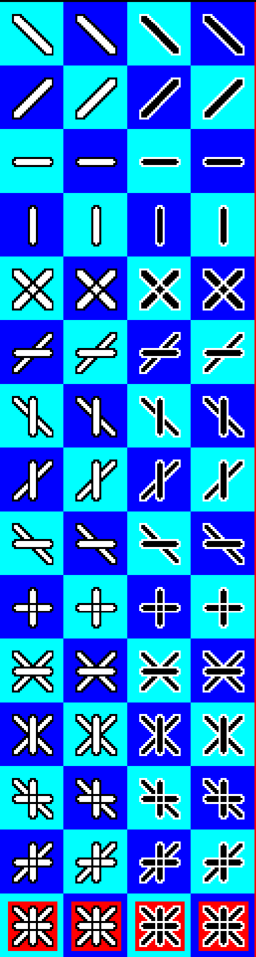
Accordingly, all sliders are rated in terms of their ratio between weighted, effective directions of attack and total directions of attack as adapted into a formula. Multiplicative factors specific to all 15 possible sliders were easily calculated. They can be readily applied to <u>any</u> 2-player chess variant <u>where the players' armies occupy the north and south sides of the board</u> at the start of the game.

For convenience, the following table of pre-calculated values is provided ready for use:

3-directional foci factors for sliders

pieces	3-dir foci factor (decimal)	3-dir foci factor (fraction)
diagon I	$1.\overline{1}$	10/9
diagon II	$1.\overline{1}$	10/9
horizon	$0.\overline{4}$	4/9
verizon	$1.\overline{3}$	4/3
bishop	$1.\overline{1}$	10/9
zig-zag	$0.\overline{7}$	7/9
zag-zig	$1.\overline{2}$	11/9
zag-zag	$1.\overline{2}$	11/9
zig-zig	$0.\overline{7}$	7/9
rook	$0.\overline{8}$	8/9
horizon-bishop	$0.\overline{8}$	8/9
verizon-bishop	$1.\overline{185}$	32/27
diagon-rook I	$0.\overline{962}$	26/27
diagon-rook II	$0.\overline{962}$	26/27
queen	1.000	1

Piece Set



2	Diagon I
2	Diagon II
2	Horizon
2	Verizon
4	Bishop
4	Zig-Zag
4	Zag-Zig
4	Zag-Zag
4	Zig-Zig
4	Rook
6	Horizon-Bishop
6	Verizon-Bishop
6	Diagon-Rook I
6	Diagon-Rook II
8	Queen

piece	diagon I

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	3

total directions	2
total directions factor	1
total directions rating	2

vital directions rating ------------------------------ = x total directions rating	3/2
x + 1 = y	5/2
weighted average = z [“y” for all 15 sliders]	9/4
y/z = 3-directional foci factor	decimal: $1.\overline{1}$ fraction: 10/9

piece	diagon II

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	3

total directions	2
total directions factor	1
total directions rating	2

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------------------------------ = x
total directions rating</td><td colspan="2">3/2</td></tr>
<tr><td>x + 1 = y</td><td colspan="2">5/2</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td colspan="2">9/4</td></tr>
<tr><td rowspan="2">y/z = 3-directional foci factor</td><td>decimal</td><td>fraction</td></tr>
<tr><td>$1.\overline{1}$</td><td>10/9</td></tr>
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piece	horizon
vital diagonal directions	0
vital diagonal factor	3
vital diagonal rating	0
vital vertical directions	0
vital vertical factor	4
vital vertical rating	0
vital directions rating	0
total directions	2
total directions factor	1
total directions rating	2
vital directions rating ------------------------------ = x total directions rating	0
x + 1 = y	1
weighted average = z [“y” for all 15 sliders]	9/4
y/z = 3-directional foci factor	decimal: $0.\overline{4}$ fraction: 4/9

piece	verizon

vital diagonal directions	0
vital diagonal factor	3
vital diagonal rating	0

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	4

total directions	2
total directions factor	1
total directions rating	2

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------------------------------ = x
total directions rating</td><td>2</td></tr>
<tr><td>x + 1 = y</td><td>3</td></tr>
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[“y” for all 15 sliders]</td><td>9/4</td></tr>
<tr><td>y/z = 3-directional foci factor</td><td><table><tr><td>decimal</td><td>fraction</td></tr><tr><td>$1.\overline{3}$</td><td>4/3</td></tr></table></td></tr>
</table>

piece	bishop

vital diagonal directions	2
vital diagonal factor	3
vital diagonal rating	6

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	6

total directions	4
total directions factor	1
total directions rating	4

vital directions rating ------------------------------ = x total directions rating	3/2
x + 1 = y	5/2
weighted average = z [“y” for all 15 sliders]	9/4
y/z = 3-directional foci factor	decimal: $1.\overline{1}$ fraction: 10/9

piece	zig-zag

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	3

total directions	4
total directions factor	1
total directions rating	4

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total directions rating</td><td>3/4</td></tr>
<tr><td>x + 1 = y</td><td>7/4</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td>9/4</td></tr>
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</table>

piece	zag-zig

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	7

total directions	4
total directions factor	1
total directions rating	4

vital directions rating ------------------------------ = x total directions rating	7/4
x + 1 = y	11/4
weighted average = z [“y” for all 15 sliders]	9/4
y/z = 3-directional foci factor	decimal: $1.\overline{2}$ fraction: 11/9

piece	zag-zag

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	7

total directions	4
total directions factor	1
total directions rating	4

vital directions rating ------------------------------ = x total directions rating	7/4
x + 1 = y	11/4
weighted average = z [“y” for all 15 sliders]	9/4

	decimal	fraction
y/z = 3-directional foci factor	$1.\overline{2}$	11/9

piece	zig-zig

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	3

total directions	4
total directions factor	1
total directions rating	4

vital directions rating ------------------------------ = x total directions rating	3/4	
x + 1 = y	7/4	
weighted average = z [“y” for all 15 sliders]	9/4	
y/z = 3-directional foci factor	decimal	fraction
	$0.\overline{7}$	7/9

piece	rook

vital diagonal directions	0
vital diagonal factor	3
vital diagonal rating	0

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	4

total directions	4
total directions factor	1
total directions rating	4

<table>
<tr><td>vital directions rating
------------------------------ = x
total directions rating</td><td>1</td></tr>
<tr><td>x + 1 = y</td><td>2</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td>9/4</td></tr>
<tr><td>y/z = 3-directional foci factor</td><td><table><tr><th>decimal</th><th>fraction</th></tr><tr><td>$0.\overline{8}$</td><td>8/9</td></tr></table></td></tr>
</table>

piece	horizon-bishop

vital diagonal directions	2
vital diagonal factor	3
vital diagonal rating	6

vital vertical directions	0
vital vertical factor	4
vital vertical rating	0

vital directions rating	6

total directions	6
total directions factor	1
total directions rating	6

<table>
<tr><td>vital directions rating
------------------------------ = x
total directions rating</td><td>1</td></tr>
<tr><td>x + 1 = y</td><td>2</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td>9/4</td></tr>
<tr><td>y/z = 3-directional foci factor</td><td><table><tr><td>decimal</td><td>fraction</td></tr><tr><td>$0.\overline{8}$</td><td>8/9</td></tr></table></td></tr>
</table>

piece	verizon-bishop

vital diagonal directions	2
vital diagonal factor	3
vital diagonal rating	6

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	10

total directions	6
total directions factor	1
total directions rating	6

<table>
<tr><td colspan="1">vital directions rating
------------------------------ = x
total directions rating</td><td colspan="2">5/3</td></tr>
<tr><td>x + 1 = y</td><td colspan="2">8/3</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td colspan="2">9/4</td></tr>
<tr><td rowspan="2">y/z = 3-directional foci factor</td><td>decimal</td><td>fraction</td></tr>
<tr><td>$1.\overline{185}$</td><td>32/27</td></tr>
</table>

piece	diagon-rook I

vital diagonal directions	1
vital diagonal factor	3
vital diagonal rating	3

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	7

total directions	6
total directions factor	1
total directions rating	6

vital directions rating ------------------------------ = x total directions rating	7/6
x + 1 = y	13/6
weighted average = z [“y” for all 15 sliders]	9/4

	decimal	fraction
y/z = 3-directional foci factor	$0.\overline{962}$	26/27

piece	**diagon-rook II**

vital diagonal directions	**1**
vital diagonal factor	**3**
vital diagonal rating	**3**

vital vertical directions	**1**
vital vertical factor	**4**
vital vertical rating	**4**

vital directions rating	**7**

total directions	**6**
total directions factor	**1**
total directions rating	**6**

vital directions rating **------------------------------ = x** **total directions rating**	**7/6**	
x + 1 = y	**13/6**	
weighted average = z **[“y” for all 15 sliders]**	**9/4**	
y/z = 3-directional foci factor	**decimal**	**fraction**
	$0.\overline{962}$	**26/27**

piece	queen

vital diagonal directions	2
vital diagonal factor	3
vital diagonal rating	6

vital vertical directions	1
vital vertical factor	4
vital vertical rating	4

vital directions rating	10

total directions	8
total directions factor	1
total directions rating	8

<table>
<tr><td>vital directions rating
------------------------------ = x
total directions rating</td><td colspan="2">5/4</td></tr>
<tr><td>x + 1 = y</td><td colspan="2">9/4</td></tr>
<tr><td>weighted average = z
[“y” for all 15 sliders]</td><td colspan="2">9/4</td></tr>
<tr><td rowspan="2">y/z = 3-directional foci factor</td><td>decimal</td><td>fraction</td></tr>
<tr><td>1</td><td>1</td></tr>
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supreme piece(s) enhancements
[selective adjustments]

Having a superior number of supreme pieces is a potentially game-winning advantage in the endgame. Consequently, it is important to remain cognizant bilaterally of the number of supreme pieces you possess compared to your opponent so that you do not end-up in a predicament where your opponent is the only player who possesses the most powerful piece(s) on the board. After all, the entire purpose of the "supreme piece(s) enhancements" is to help prevent exactly the described predicament.

There are three significant, measurable types of "supreme piece(s) enhancements": the "non color-bound enhancement", the "non color-changed enhancement" and the "compound enhancement". All of the "supreme piece(s) enhancements" should be totaled before they are applied to the unadjusted practical attack values of the relevant pieces.

The "non color-bound enhancement" is weighted at exactly twice that of the "non color-changed enhancement". This 2:1 ratio is due to the fact that the color-bound problem, left uncorrected, would exist on <u>all</u> consecutive moves with a bishop while the color-changed problem, left uncorrected, would exist only on alternating moves (i.e., ½ of the moves) with a knight.

non color-bound enhancement
[supreme piece(s) enhancements]
[selective adjustments]

In Chess (for example), it is well-established that two bishops distributed on opposite spaces (light and dark) are individually a little more effective and valuable per bishop than one bishop on either light or dark spaces. This is due to the color-bound nature of bishops (i.e., light or dark spaces exclusively).

Of course, any decently designed chess variant will have one or more pairs of bishops balanced upon opposite spaces (light and dark) at the opening setup. In Chess (for example), there is one bishop upon light spaces and one bishop upon dark spaces per player at the start of the game. It is upon this basis that the material values of the bishops are calculated within this work. However, if/when either one of the two bishops is captured, then one player no longer has any bishops present upon either the light or dark spaces. Accordingly, the material value (unadjusted practical attack value, to be exact) of the remaining one bishop should be reduced slightly.

Therefore, a "color-bound penalty" of appr. 11.11% (8/9 of its original, unadjusted practical attack value) should be applied in any chess variant to every remaining bishop of a player whenever the bishops no longer have a presence upon both the light and dark spaces (i.e., bishops are present exclusively upon either light <u>or</u> dark spaces but not both).

It is obviously a significant advantage for any single piece that possesses, in part, the movement capabilities of a bishop to NOT be color-bound and to NOT possibly have its unadjusted practical attack value reduced moderately if its counterpart piece upon opposite spaces (light or dark) is captured. Of course, this requires the piece to have color-changing movement capabilities as well.

Accordingly, the "non color-bound enhancement" applies to <u>all</u> pieces that have a bishop component in conjunction with some other movement capability to escape being color-bound.

With an archbishop, the addition of the knight component makes <u>all</u> of its possible knight moves to other spaces effective at breaking the color-bound tendency attributable to its bishop component. This merits a 25.00% (1/4) bonus <u>per piece</u> applied to its unadjusted practical attack value.

With a queen, the addition of the rook component makes exactly <u>½</u> of its possible rook moves to other spaces effective at breaking the color-bound tendency attributable to its bishop component. This merits a 12.50% (1/8) bonus <u>per piece</u> applied to the unadjusted practical attack value.

Note that the "color-bound penalty" of appr. 11.11% (8/9) and "non color-bound enhancement" of 12.50% (9/8) for ½-effective color-changing are multiplicative inverses of one another.

non color-changed enhancement
[supreme piece(s) enhancements]
[selective adjustments]

It is also a slight advantage for any single piece that possesses, in part, the movement capabilities of a knight to NOT be forced to change colors. Of course, this requires the piece to have color-binding movement capabilities as well.

Accordingly, the "non color-bound enhancement" applies to <u>all</u> pieces that have a knight component in conjunction with some other movement capability to prevent being color-changed.

With an archbishop, the addition of the bishop component makes <u>all</u> of its possible bishop moves to other spaces effective at breaking the color-changing tendency attributable to its knight component. This merits a 12.50% (1/8) bonus <u>per piece</u> applied to its unadjusted practical attack value.

With a chancellor, the addition of the rook component makes exactly <u>½</u> of its possible rook moves to other spaces effective at breaking the color-changing tendency attributable to its knight component. This merits a 6.25% (1/16) bonus <u>per piece</u> applied to the unadjusted practical attack value.

compound enhancement
[supreme piece(s) enhancements]
[selective adjustments]

In Chess (for example), anyone who understands the game well would warn that, all things otherwise equal, it is a mistake to exchange your 1 queen for 1 rook and 1 bishop belonging to your opponent. Indeed it is despite the fact that it seems logical to expect the obvious- for the material values to be such that 1 queen is exactly equal to the combined value of 1 rook and 1 bishop.

In Chess where each player starts the game with only 1 queen, when 1 queen is exchanged for 1 rook and 1 bishop, (except in occasional games where at least one pawn has been promoted into a queen) the player who gave-up the queen has either:

1. **given-up the advantage of being the only player with the most powerful piece on the board.**

OR

2. **made it so that his/her opponent has the advantage of being the only player with the most powerful piece on the board.**

Normally- If you are the only player in Chess who has a queen, then you have a decisive, irrefutable, long-term positional advantage as a direct consequence that, properly managed throughout the course of the game, will probably, eventually lead to your victory.

The nature of the advantage resides within the superior forking ability of the most powerful piece and the inevitable dilemma faced in protecting all pieces from theft even as the most powerful piece can be moved a few times consecutively in such a manner that each offensive move (esp. “checks”) requires a defensive move (by pieces with inferior forking ability) to prevent an immediate theft of a piece.

During the rest of the game, a position will often eventually arise where a multiple move by the most powerful piece will cause a dilemma resulting irrefutably in a theft of a piece. Once understood in these tactical and/or strategic terms, the “compound enhancement” no longer appears mysterious, arbitrary or contrived at all.

An appropriate, definable material value can be ascribed to this long-term positional advantage and in fact, is a needed adjustment to create active deterrence rather than passive neutrality toward accepting the disadvantageous end of this exchange. Most chess variants require a similar adjustment for usable material values to be possible.

I hate to intentionally misclassify what I know correctly to be a "long-term positional value" as a "material value adjustment", instead.

The reason I am willing to do so (in this case only) is that I am sure of my mathematical assessment that chess variants will remain intractible in their midgames (where opening books and endgame tablebases cannot be used) to computer chess programs even when futuristic advances in computer technology (especially CPU speeds) and AI programming are allowed for.

This means that ALL computer chess programs today, even when running at the highest time or depth controls on state-of-the-art hardware, will never be able to compute far enough into the future of the game to FULLY credit positional values of a long-term nature. Instead, they will reach and get stuck within their deepest attainable ply (by any survivable measure of time) where a combinatorial explosion occurs, leaving positional values of a long-term nature only PARTIALLY credited at most.

In other words, computer chess programs have the fundamental limitation of being strictly tactical and not strategic at all since they are unable to address important endgame issues while computing, esp. within the opening game. Consequently, I think the only way to assure that the long-term positional values of supreme pieces are fully accounted for is to artificially introduce them as an enhancement into material values by an appropriate amount.

enhancements sum
[selective adjustments]

Experimentation using computer chess programs has established that an enhancement sum of appr. 18.75% (3/16) works best for the queen in the game of Chess. An identical quantitative extrapolation from Chess to a large variety of chess variants is tentatively used until/unless more accurate values are discovered for individual games. Thus, a system appropriately using the sum of any or all of the "non color-bound enhancement", the "non color-changed enhancement" and the "compound enhancement" is to be applied to the unadjusted practical attack values of all eligible pieces within all chess variants.

In the example of Chess …

The 1 queen per player the game starts with is the only piece that is eligible for the "non color-bound enhancement" effective only upon ½ of the spaces. The 12.50% (1/8) bonus per piece applies only to it.

The 1 queen per player the game starts with is the only piece that is eligible for the "compound enhancement". The 6.25% (1/16) bonus per piece applies only to it.

The sum of the "non color-bound enhancement" and the "compound enhancement" in Chess involves one piece:

For the queen, it is an 18.75% bonus.

The (adjusted) practical attack value for the 1 queen is 98.92 (83.30 X 1.1875).

Thereby the table of practical attack values can be completed.

In the example of Embassy Chess …

The 1 archbishop and 1 queen per player the game starts with are the only 2 pieces that are eligible for the "non color-bound enhancement".

The 25.00% (1/4) bonus per piece applies to the archbishop which is effective upon all of the spaces.

The 12.50% (1/8) bonus per piece applies to the queen which is effective only upon ½ of the spaces.

The 1 archbishop and 1 chancellor per player the game starts with are the only 2 pieces that are eligible for the "non color-changed enhancement".

The 12.50% (1/8) bonus per piece applies to the archbishop which is effective upon all of the spaces.

The 6.25% (1/16) bonus per piece applies to the chancellor which is effective only upon ½ of the spaces.

The 1 archbishop, 1 chancellor and 1 queen per player the game starts with are the 3 pieces that are eligible for the "compound enhancement" of 6.25% (1/16) bonus per piece.

The sum of the "non color-bound enhancement", the "non color-changed enhancement" and the "compound enhancement" in Embassy Chess involves three pieces:

For the archbishop, it is a 43.75% bonus.
For the chancellor, it is a 12.50% bonus.
For the queen, it is a 18.75% bonus.

The (adjusted) practical attack value for the archbishop is 98.22 (68.33 X 1.4375).
The (adjusted) practical attack value for the chancellor is 101.48 (90.20 X 1.1250).
The (adjusted) practical attack value for the queen is 115.18 (96.99 X 1.1875).

Thereby the table of practical attack values can be completed.

how to calculate practical attack values

1. **Make sure that all ideal attack values from various tables for different classes of pieces that are not already on par with one another are adjusted to be directly comparable, baseline values defined tangibly by the average number of spaces that pieces can attack upon a given board.**

2. **Have the ideal attack values for all pieces of unlimited range (i.e., sliders) on hand.**

3. **Multiply the required 1-4 selective move blocks by the ideal attack values for each piece of unlimited range (i.e., slider) individually to obtain 1-4 products per slider.**

   IF only 1 selective move block is required, then skip steps #4 & #5.

4. **Add all 1-4 products together to obtain their total.**

5. **Divide their total by the number of selective move blocks comprising their total to obtain their average.**

   Repeat until the averages for every piece of unlimited range (i.e., slider) are obtained.

6. **Multiply the averages by the 3-directional foci factors for every piece of unlimited range (i.e., slider).**

   [Note- Use the table on page 21.]

   Migrate them into the table of unadjusted practical attack values.

   IF pieces of unlimited range (i.e., sliders) are exclusively used in the game, then skip steps #7 & #8.

7. **If any pieces with limited range are used, have the ideal attack values for all pieces on hand.**

8. **Since ideal attack values are already equivalent to unadjusted practical attack values for pieces with limited range, just migrate them numerically-unaltered into the table of unadjusted practical attack values.**

9. Calculate the “non color-bound enhancement” for all eligible pieces by using the examples appropriate to the game at hand.

10. Calculate the “non color-changed enhancement” for all eligible pieces by using the examples appropriate to the game at hand.

11. Calculate the “compound enhancement” for all eligible pieces by using the examples appropriate to the game at hand.

12. Calculate the enhancements sum of the “non color-bound enhancement”, the “non color-changed enhancement” and the “compound enhancement” for all eligible pieces. Apply them to the relevant pieces within the table of unadjusted practical attack values to obtain the table of adjusted practical attack values.

13. Compare all values. Find the neatest available empirical ratio for all pieces (to an accuracy of 1/100 of a point) where the least-valued piece equals exactly 10 points. This is the polished table of practical attack values.

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royal values, ideal and practical

Ideal royal values are variables determined by estimation.
Practical royal values are constants determined by estimation.

Royal values (ideal and practical) are always game-specific and never board-specific.

Up to this point, calculations have been used exclusively.
Beyond this point, estimates are required and compounded.

A royal piece(s) usually [but not always!] has a nominal, practical attack value yet a supremely-high, practical royal value with its total material value being an even higher sum. In the special case where a royal piece(s) has zero practical attack value, then its total material value is perfectly interchangeable with its practical royal value.

army values
[ideal royal values]

Before ideal royal values can actually be estimated, a series of computations must be performed to calculate needed values. The army value calculations are quickly and easily obtained with basic knowledge of the game at hand as it involves only visual observation and simple, previously-covered methods.

how to calculate army values

1. **Take an enemy army (i.e., opening setup) from the start of the game.**

2. **Calculate the practical attack values for every individual piece in the enemy army (except its royal piece).**

3. **Add-up the practical attack values for every individual piece in the enemy army (except its royal piece) to obtain the total. This is the first army value.**

4. **Repeat this cycle of calculation every time an enemy piece is captured or promoted.**

ideal exchange deterrent

Although noone has ever presented a formal, systematic calculation method for deriving provably-accurate, material values for a royal piece(s) that is universal (applicable to any given chess variant), a few experts have proposed and implemented an expedient, systematic method for deriving approximate, realistic material values which work reasonably-well both for AI programs and rational minds of human players. In modern times, reliable estimates can be refined thru computer and/or human playtesting.

Originally tailored to Chess, the main idea was to value the royal piece materially at significantly more than the combined, material values of ALL of the opponent's other pieces except the royal piece(s),

By such a scheme, any sacrifice or exchange involving the only or last royal piece would be avoided at all costs. Reportedly, a multiplicative factor of 1.125 or 9/8 works well as appropriate, measured deterrence in single royal piece games via computer chess. By the way, if the factor is too high, the program plays with a tendency to waste moves trivially improving the safety of the royal piece. Accordingly, the factor representing the ideal exchange deterrent in this system is set at exactly 1.125 or 9/8 as well. Note that since this calculation must be repeated with every capture or promotion of an enemy piece, the ideal royal values it yields are variable (determined many times) instead of constant (determined once at the start of the game).

ideal exchange deterrent = 1.125

ideal royal values =

army values X ideal exchange deterrent =

army values X 1.125

royal overvaluation corrections
[practical exchange deterrent]

1. "replaceability of royal pieces"

A "yes" that royal pieces can be replaced via promotion will make royal overvaluation corrections necessary.

A "no" that royal pieces cannot be replaced via promotion will make royal overvaluation corrections unnecessary.

Still, this is more than just a "yes" or "no" conditional.

If the royal piece(s) are replaceable, then this "yes" condition needs to be qualified and estimated via numerous game-specific criteria. Generally, the higher the replaceability, the lower the multiplicative factor. Some considerations are …

- **A. How many pieces are illegible for promotion to replace the royal piece(s)?**
- **B. What fraction of the total army are the pieces illegible for promotion to replace the royal piece(s)?**
- **C. How many moves does each piece require for promotion?**
- **D. What skill level is required to achieve promotion(s)?**
- **E. What are the realistic odds of achieving promotion(s) against a vigilant opponent under typical game conditions?**
- **F. How many promotions per game can a resourceful player expect to achieve?**
- **G. Do other vital aspects of responsibly managing the game have to be sacrificed to achieve promotion(s)?**

2. “irreplaceability of piece numbers”

A “yes” that the number of pieces upon the board is permanently, irreversibly decreased with each capture will make royal overvaluation corrections necessary.

A “no” that the number of pieces upon the board, although it can be temporarily decreased with each capture, can also be reversibly increased via some method will make royal overvaluation corrections unnecessary.

This is just a “yes” or “no” conditional.

If piece numbers are irreplaceable, then this “yes” condition needs to be qualified and estimated as a game-specific criterion. However, this condition is an absolute that either applies or it does not. In either case, there are no details to investigate.

3. “practical attack values of royal piece(s)”

A “yes” that the royal piece(s) have practical attack values will make royal overvaluation corrections necessary.

A “no” that the royal piece(s) do not have any practical attack values (i.e., the royal pieces are immobile) will make royal overvaluation corrections unnecessary.

Still, this is more than just a “yes” or “no” conditional.

If the royal piece(s) have non-zero, practical attack values, then this “yes” condition needs to be qualified and estimated as a game-specific criterion. Generally, the higher the practical attack values, the lower the multiplicative factor. By the way, the figure for this piece(s) has already been calculated in a previous step. Unfortunately, there is no established translation function between practical attack values and the multiplicative factor representing royal overvaluation correction #3.

Reliably estimating this condition starts with assessing whether the royal piece(s) has a relatively low, medium or high practical attack value compared to the average of all other, non-royal pieces.

Unfortunately, all 3 multiplicative factors representing the 3 royal overvaluation corrections are of a purely estimative and game-specific nature that have defeated all of my attempts thusfar at "universal formula finding". Therefore, no universal method for achieving reliable estimates across a variety of games is known. Still, I think I was marginally successful at devising a reliable method customized for one game I invented (albeit with much time, effort and playtesting).

Fortunately, methods for testing the reliability of the combined estimate of royal overvaluation corrections exist.

If one is very lucky, none of the 3 factors require estimation since "no" was the answer to all 3 conditions. If one is very unlucky, all 3 factors require estimation since "yes" was the answer to all 3 conditions.

Although the values for each of the 3 multiplicative factors can vary greatly (0-1) and as a result, the combined value of the royal overvaluation corrections can also vary greatly (0-1), reliable estimates seem to be achievable with careful, thorough analysis. In any case, refinements to within-range values can definitely be made via computer and/or human playtesting.

[Important! Note that an inverse scale is defined whereby the royal overvaluation corrections are described as highest where the value of zero is approached and lowest where the value of 1 is approached.]

Always use tangible methods with a proven, solid connection to reality instead of purely theoretical thought lacking feedback to make refinements. Otherwise, your imagined refinements could backfire, definitively being degradations instead. Just be mindful that playtesting examples should never be taken to destabilizing extremes where they may prove nothing.

Admittedly, some types of chess variants would play terribly via computer chess AI if grossly-unreliable estimates for any of the 3 factors were provided at this critical juncture while others would still play fine- without even taking royal overvaluation corrections into account.

Only if a person has incisive, game-specific information and a good work ethic can a reliable, well-defined material value for the royal piece(s) in the game that is important to him/her eventually be reached (that within-range estimates of the royal overvaluation corrections made herein predetermined).

It is useless to belabor the fact that the critically-important royal overvaluation corrections are of a labor-intensive, game-specific nature and that, consequently, I can only provide a few general guidelines lacking in mathematical detail.

practical exchange deterrent =

ideal exchange deterrent X royal overvaluation corrections =

1.125 X royal overvaluation corrections

---

practical royal values =

ideal royal values X practical exchange deterrent

For two sharply contrasting examples …

1. **The royal overvaluation corrections for Chess, where the "replaceability of royal pieces" is "no", the "irreplaceability of piece numbers" is "yes" and the "practical attack values of royal piece(s)" is "yes" are extremely low (with the product of all 3 factors estimated at virtually "1.0"), rendering a practical exchange deterrent of virtually "1.125" (virtually equal to the ideal exchange deterrent of 1.125) within-range, readily and safely usable.**

2. **The royal overvaluation corrections for Spherical Chess 324, where all 3 conditions are "yes", are very high (with the product of all 3 factors estimated at "1/8" or "0.125"), rendering a practical exchange deterrent of "9/64" or "0.140625" within-range, readily and safely usable.**

There are two main reasons that the practical exchange deterrents are so radically divergent between the 2 example games:

1. **Despite the fact that both games share a "yes" to two conditions, "irreplaceability of piece numbers" and "practical attack values of royal piece(s)", the "yes" which applies to the "practical attack values of royal piece(s)" is very low for Chess and extremely high for Spherical Chess 324.**

2. **The "replaceability of royal pieces" condition is a "no" for Chess yet a "yes" for Spherical Chess 324 that applies extremely high.**

Nonetheless, these two reasons, however satisfactory, utterly fail to provide any desired useful, numerical estimate for the needed factors (of great impact).

material values

Material values are constants determined by estimation.
Material values are always game-specific and never board-specific.

Practical attack values are constants determined by calculation.
Practical royal values are variables determined by estimation.

Material values are the sum of the component practical attack values and practical royal values (if/when applicable) of pieces. They are indeed perfectly interchangeable with practical attack values except in the critically-important special case of a royal piece(s). Notwithstanding, this special case is the designated game-winning condition in common with most chess variants.

Although this is admittedly an incomplete formulation (to date), material values are generally the most efficacious component of relative piece values and as such, are a sound foundation as well as a reliable starting point toward a complete, more complex formulation (if needed).

The limitations to the accuracy of material values as a foundation for ultimately determining relative piece values across a wide variety of games, boards and pieces are not a serious problem since this framework is open-ended to allow all needed and game-specific refinements (of virtually any conceivable conceptual and numerical nature) to be appropriately input, applied, properly-weighted and calculated in the overlying layer.

The proper classification, organization and isolation of all of the most efficacious, important factors (albeit some are expressed only in general terms) that determine relative piece values has been provided at this point, nonetheless.

Please be forewarned that faulty methods [often conceptual errors instead of obvious or non-obvious mathematical errors] for performing the complex estimates or calculations required for a complete formulation of relative piece values litter the chess variant and board game literature, on and off the internet.

Only refinements that an intelligent person firmly understands and is convinced of the conceptual validity and numerical accuracy of should be made. Otherwise, the “crude, simplistic, incomplete” material values attained here by routine means that one desired to improve upon could definitively be less inaccurate than the “sophisticated, complex, complete” relative piece values attained elsewhere thru much hard work.

A refinement of probable merit not included within this work is to compare the total material values of the pieces of both players every time there is an exchange or capture. Of course, the total material values of the pieces of both players are equal at the start of the game. Nonetheless, as soon as exchanges or captures begin, the likelihood of creating a material imbalance increases. At some point within most games, a material imbalance will exist.

For example …

If white has a total of 1000 points worth of pieces remaining while black has a total of 900 points worth of pieces remaining, then white has a 10:9 ratio advantage over black materially. This gives white the prerogative of forcing exchanges of pieces with equal material to its greater advantage materially. Specifically, if white forces the exchange of two identical pieces worth 100 points (for instance) with black, then white will have a total of 900 points worth of pieces remaining while black will have a total of 800 points worth of pieces remaining for a 9:8 ratio advantage.

Notably, this 9:8 ratio advantage is even greater than the previous 10:9 ratio advantage. Therefore, it is advantageous to white to play aggressively although this player must always remain wary of positional disadvantages that can be attained via playing rashly and without careful forethought. Generally, it is difficult for black to position its pieces in such a way as to prevent white from being able to force exchanges of pieces of equal material value.

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material values =

practical attack values + practical royal values

With an estimated, total material value for the royal piece(s) in place,
the landscape of material values of all pieces used within a given game is finally complete.

It is noteworthy that the variable, practical royal value of royal piece(s) actually applies to the set of royal pieces instead of to its elements. In other words, unless an enemy piece is captured or promoted, the variable representing the practical royal value of the set of royal pieces remains the same regardless of its number of elements (which changes during the course of a game).

Where there is only one royal piece, it has a variable, practical royal value that is perfectly interchangeable with the variable, practical royal value of the set of royal pieces. However, in the special case where there is more than one royal piece, each royal piece has a variable, practical royal value, being worth its fraction of the variable, practical royal value of the set of royal pieces, that increases inversely as the number of royal pieces remaining decreases due to captures.

examples of material values

Only Fischer Random Chess definitely has very well-established material values for its pieces. Nearly all other chess variants with published piece values [There are only a half a dozen.] are of dubious reliability, most with their methods of calculation, estimation or guesswork unpublished and unknown to me. Still, it definitely appears to be the case that the material values provided by a few experts for one other game, Capablanca Random Chess, are reasonably accurate. Nonetheless, these are the only two testbeds with provably-reliable material values. Anyway, to attempt to test and refine the soundness of this method, alternative material values have been calculated and provided for these two games as well as one of my own invention that does not yet have provably-reliable material values. Thusfar, for only these three games of varying popularity:

Spherical Chess 324
http://www.symmetryperfect.com/shots/texts/values-spherical.pdf

Fischer Random Chess
(including Chess)
http://www.symmetryperfect.com/shots/texts/values-chess.pdf

Capablanca Random Chess
(including Embassy Chess)
http://www.symmetryperfect.com/shots/texts/values-capa.pdf

Generally, the range of material values for pieces attained by my method is not unusual compared at least, to those responsibly calculated by others.

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positional values

Positional values of pieces are variables determined by estimation.
Positional values are always game-specific, never board-specific.

Positional values are plus or minus adjustments to the total relative piece values. In the imaginary, simplified, special case where they equal a multiplicative factor of exactly 1, relative piece values are indeed perfectly interchangeable with material values.

Of course, a game where all positional values always equal 1 cannot exist since positional values entail estimates which are not only game-specific but moreover, specific-to-the-game-state (including the exact positions of every piece for the specific game in progress). Sound, proven methods to reliably estimating positional values are generally understood by computer chess AI experts but in practice, are managed only by the sophisticated programs they develop.

Reinhard Scharnagl has explained that positional values are used within SMIRF as multiplicative inverses of the material values of the pieces and applied only to squares that are attacked or contested by one or both players during a game in progress. Generally, the playing strength of his SMIRF program attests well to the soundness of his quite-possibly brilliant method.

Despite their elusive nature, positional values become exclusively important and render material values meaningless whenever game-winning conditions are attainable. Usually, this does not occur naturally until the endgame but a serious error by one player can make its potential become dangerously real even within the opening game or midgame. Generally, material values are of greatest importance at the start of the game and positional values are of greatest importance at the end of the game with a shift gradually occurring by increments with each move in the game.

relative piece values

Relative piece values are variables determined by estimation.
Relative piece values are always game-specific, never board-specific.

Material values are constants determined by estimation.

Positional values of pieces are variables determined by estimation.

Relative piece values are the most holistic values- a complex, weighted estimate of the component material and positional values of pieces that I normally defer to a computer chess program for. After all, nothing less than an evaluation function run by a sophisticated computer program can easily estimate these values. Incidentally, they vary during gameplay and are inaccurate to the extent an AI program is limited in playing strength.

relative piece values =

material values X positional values