# the perfect symmetry number theory

## model I (a)

**abstract-**

**"the perfect symmetry number theory"**

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**In accordance with formalism, one of the two most widely accepted foundations for modern mathematics, an experimental axiomatic system having a variant number theory is admissible for study if it is self-consistent. Nonetheless, any given "revised" system is without exceptional theoretical value or applicability unless it is comparatively advantageous to the "conventional" system.**

**This unconventional work initially involves the creation of a revised multiplication in which the revised product of two negative, real number factors equals a negative real number, contrary to conventional multiplication. This precludes the existence and need for the unit imaginary number and thus, the complex number system, etc.**

**By a method analogous to how conventional involution is built upon conventional multiplication, likewise is revised involution built upon revised multiplication. Although addition is identical under both systems, with two of its three binary operations revised, a revised arithmetic exists and consequently, a revised algebra. Further ramifications include a revised analytic geometry, revised analytic trigonometry and revised calculus. In fact, every branch of mathematics that is wholly or partially based upon numerical definitions and methods is affected.**

**Comparatively, revised arithmetic requires three number systems instead of an infinite number out of which only 13 have been invented to date (i.e., no complex [2-D] or hypercomplex number systems: 4-D, 8-D, 16-D, 32-D, 64-D, 128-D, 256-D, 512-D, 1024-D, etc) and three binary operations instead of six (i.e., no inverse binary operations: subtraction, division, evolution) yet revised algebra based upon it maintains all comparable problem-solving capabilities.**

**In revised algebra, a binomial, linear equation to <u>any</u> degree is solvable since after revised cross-multiplication, it is reducible to the original, first degree equation. In conventional algebra, a binomial, linear equation to the fifth degree or higher is generally impossible to derive solutions for.**

**Ultimately, the two numerical systems are fully isomorphic in describing the same underlying mathematical reality as it exists independent of any contrasting, arbitrarily-invented, mathematical languages of interpretation but the revised system is vastly superior to the conventional system in accordance with Occam's razor.**

**the perfect symmetry number theory**
**http://www.symmetryperfect.com web site**

**the revised, extended real number system applied to arithmetic, algebra and analytic geometry**

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**Questions, comments, ideas, critiques and advice are welcome.**

**2010 MSC**

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**This paper lies primarily within the 2010 Mathematical Subject Classification 03C62 “Models Of Arithmetic & Set Theory”. Notwithstanding, it does not completely fit into 03C62 since more branches of mathematics than arithmetic are constructed. To be sure, it completely falls under the section “Model Theory” (MSC 03Cxx). One is not required to be a specialist in “Mathematical Logic & Foundations” (MSC 03-XX) to meaningfully study this work. It can also be directly approached from the viewpoint of “Number Theory” (MSC 11-XX) or any of several other areas it addresses. After all, each and every area of mathematical training carries its own characteristic insights toward the entirety of revised mathematics in a likewise manner as it does for conventional mathematics.**

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**the perfect symmetry number theory**
**model I demonstration programs**

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**unary operations**
**http://www.symmetryperfect.com/demo/functions**

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**revised arithmetic- model I**
**http://www.symmetryperfect.com/demo/I**

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**revised logarithms**
**http://www.symmetryperfect.com/demo/logs**

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**the perfect symmetry number theory**
**model I graphs (13)**
**http://www.symmetryperfect.com/graphs/1-graphs.pdf**

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**the extended real number continuum**
**circular depiction**
**http://www.symmetryperfect.com/graphs/I/1-cont-circ.pdf**

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**the extended real number continuum**
**linear depiction**
**http://www.symmetryperfect.com/graphs/I/1-cont-linear.pdf**

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**the extended real number continuum**
**circular-linear depiction**
**http://www.symmetryperfect.com/graphs/I/1-cont-dual-1.pdf**

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**the extended real number continuum**
**linear-circular depiction**
**http://www.symmetryperfect.com/graphs/I/1-cont-dual-2.pdf**

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**opposition**
**http://www.symmetryperfect.com/graphs/I/1-opp.pdf**

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**reciprocation**
**http://www.symmetryperfect.com/graphs/I/1-rec.pdf**

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**opposition and reciprocation**
**http://www.symmetryperfect.com/graphs/I/1-opp-rec.pdf**

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**addition**
**http://www.symmetryperfect.com/graphs/I/1-add.pdf**

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**revised multiplication**
**model I**
**http://www.symmetryperfect.com/graphs/I/1-multiply.pdf**

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**revised involution**
**model I**
**http://www.symmetryperfect.com/graphs/I/1-involute.pdf**

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**revised power functions**
**http://www.symmetryperfect.com/graphs/I/1-power.pdf**

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**revised exponential-logarithmic functions**
**x y± axes**
**model I**
**http://www.symmetryperfect.com/graphs/I/1-e-log-1.pdf**

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**revised exponential-logarithmic functions**
**x± y axes**
**model I**
**http://www.symmetryperfect.com/graphs/I/1-e-log-2.pdf**

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**"the perfect symmetry number theory"**

**(applied to arithmetic, algebra and analytic geometry)**

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**introduction**

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**The holistic structure and outline of this paper follows the objective of applying the revised, extended real number system to arithmetic, algebra and analytic geometry and in a numerically and axiomatically self-consistent manner.**

**By mathematically modeling the revised, perfect symmetry number theory in detail, therein describing its algorithms, structures and properties, a direct comparison to the conventional (broken symmetry) number theory is initiated which can be followed through and exhaustively verified by specialists in number theory, axiomatics, algorithms, etc. Moreover, one is free to extend the methodical construction of the revised system throughout analytic trigonometry, differential and integral calculus, real analysis or any desired applications in engineering sciences or mathematical physics.**

**Of course, inerrantly accomplishing and validating such works would be tedious, time-consuming and complicated. They would have permanent value, nonetheless, by enabling the revised number theory to be applied in numerous, useful ways and achieving a definitive, enriched comparison to the conventional number theory.**

**The work developed thusfar consists of four parts-**

**I. central concepts and explanations**
**II. fundamentals**
**III. revised arithmetic and revised algebra**
**IV. revised analytic plane geometry**

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**The approach used in its presentation is a methodical, foundational build-up of concepts through three branches of mathematics with an open-ended potential for further development.**

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**"central concepts and explanations"**

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**Part I consists of nine sections.**

**All sections in part I are essays, some technical, provided to add needed meanings to foundational or technical areas benefiting from explanation. Despite holding value judgments in select places, they are responsible and factual.**

**The first essay, "project overview", goes into more detail than the abstract in examining revised arithmetic and revised algebra. Then, it gives some practical justifications for the hardline position taken.**

**The second essay, "minimal completeness and maximal applicability", is an objective comparison of the revised and conventional systems of arithmetic and algebra by important, incontrovertible and measurable criteria.**

**The third essay, "comparing numerical systems", is a holistic description of the value, meaning, purpose and limitations of this project.**

**The fourth essay, "an unnatural history", chronicles key, disastrous events in the history of the development of conventional arithmetic and algebra.**

**The fifth essay, "unclear foundations of math", does not dogmatically declare a right or wrong position. Instead, it advocates the continuation of a practical approach as it relates to this project.**

**The sixth essay, "the non-absoluteness of mathematical proofs", points out the relative-axiomatic limitations of numerically-consistent, mathematical proofs when applied to two isomorphic, numerical systems.**

**The seventh essay, "an unconventional approach", explains the method chosen for presenting this work and the reasons for it.**

**The eighth essay, "representations with rectangular coordinate systems", is a refutiation of certain aspects of graph theory where conventional binary operations are represented with rectangular coordinate systems. A stricter set of rules applies unbroken in graph theory where revised binary operations are represented with rectangular coordinate systems.**

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**The ninth essay, "in search of intelligent life", is a fictionalized satire about the encounter between an alien visiting Earth to assess our level of mathematical competence and a proud, arrogant science representative. Some will not find it amusing at all but the dialog is informative.**

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**"fundamentals"**

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**Part II consists of five sections.**

**The first section, "special definitions", explains the meaning of important terms and concepts that are unique to this work.**

**The second section, "important distinctions in terminology", lists where conventional and revised counterparts do or do not require distinction.**

**The third section, "symbols", lists the mathematical symbols used within this work and what they are termed.**

**The fourth section, "the extended real number continuum", presents a universal geometrical and numerical model of the set of extended real numbers. The single reality of the model is presented variously via four depictions: circular, linear, circular-linear and linear-circular.**

**The geometric relations of every extended real number in the model under the unary operations of opposition and/or reciprocation are referenced.**

**The revised slope system is also presented with perfect correlation to the extended real number continuum. It is a foundational distinction relevant to revised analytic geometry and in turn, revised calculus.**

**The fifth section, "unary operations", presents the unary operations, opposition and/or reciprocation, in the form of functional notation.**
**The only thing new is the concise symbols introduced for these familiar unary operations that are often used and must be learned.**

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**"revised arithmetic and revised algebra"**

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**Part III consists of one section- "revised binary operations".**

**It is the most important, vital part of the paper. In fact, the heart of the perfect symmetry number theory lies within the rules of revised arithmetic generally and revised multiplication specifically. The differences which emerge within revised arithmetic, revised algebra, revised analytic geometry, revised trigonometry, revised calculus, etc. are direct or indirect consequences of this single radical departure from conventional multiplication at the foundational level. Indeed, all higher numerical or analytical structure within most branches of mathematics is strongly affected. These differences are intrinsic, necessary and unavoidable.**

**Since the general laws of revised arithmetic are expressed as simple equations in revised algebra, these two branches of mathematics are presented in this work inseparably interwoven.**

**The computational characteristics akin to revised multiplication and revised involution, contrasting conventional arithmetic, must be studied until mastered. Otherwise, any further mathematics is unlikely to be understood and any opinions developed for or against the validity of this theory are uninformed and insignificant. However, there are a few formalistic adaptations and unique, new concepts to the revised numerical system that must be dealt with- "identical multipliers/exponents", "revised logarithms" and "correspondent notation". None of these uniquenesses are a challenge to understand, though.**

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**"revised analytic plane geometry"**

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**Part IV consists of three sections.**

**It contains the essentials of this branch of mathematics that is created from the synthesis of pure geometry, which is absolutely unchanged, with revised algebra, which varies from conventional algebra. Consequently, most areas familiar to conventional analytic geometry are altered significantly under revised analytic geometry.**

**The first section, "revised linear equations", contains proofs of the vastly improved simplification and superior problem-solving capabilities for revised algebra over conventional algebra.**

**In revised analytic geometry, any binomial, linear equation (1st degree), revised cross-multiplied by itself any given number of times (to the n-th degree), is in all cases reducible to the original binomial, linear equation (1st degree) which is solvable.**

**The second section, "revised linear functions", has an analogous basic structure to the revised linear equations and moreover, is an algebraic generalization of revised linear equations.**

**The third section, "functions involving revised involution", maps indicative revised power functions and revised exponential/logarithmic functions under the revised numerical system and describes their properties.**

**A system with consistent graph-function relationships in two dimensions (plane) can be extended to three dimensions (space) as a matter of course. Accordingly, although only revised analytic plane geometry (2-D) is modeled herein for educational clarity, its validity and self-consistency insure that revised analytic solid geometry (3-D) can be modeled successfully as well.**

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# central concept and explanations

## part I

**project overview**

**This unconventional work involves initially the creation of a revised multiplication unlike conventional multiplication. By a method analogous to how conventional involution is built upon conventional multiplication, likewise is a revised involution built upon revised multiplication.**
**With two of its three binary operations revised, a revised arithmetic exists and consequently, a revised algebra.**

**In conventional algebra, there is no real number, positive or negative, multiplied by itself that equals a negative, real number product.**

**For example, using –1 …**

$$n \times n \neq -1$$

$$+1 \times +1 = +1$$
$$-1 \times -1 = +1$$

**Therefore, the concept of the unit imaginary number "i" had to be invented to solve such equations.**

**For example, using –1 …**

$$i \times i = -1$$

**Together, the real number system with the imaginary unit forms the complex number system that is also indispensable (to conventional algebra).**

**Conversely …**

**In revised algebra, any negative real number multiplied by itself equals a negative, real number product.**

**For example, using –1 …**

$$-1 \times -1 = -1$$

**This exhibits perfect, mirror-image symmetry with the indisputable fact that any positive real number multiplied by itself equals a positive, real number product (in revised algebra <u>and</u> conventional algebra).**

**For example, using +1 …**

**+1 x +1 = +1**

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**In ½ of the cases, revised multiplication yields slightly different revised products (with the same absolute values but opposite signs) compared to conventional multiplication. In ½ of the cases, revised multiplication yields identical products.**

**In ¾ of the cases, revised involution yields different revised powers compared to conventional involution. In ¼ of the cases (where both the base and exponent are positive real numbers), revised involution yields identical powers.**

**One of a few important advantages in using revised binary operations instead of conventional binary operations is that this revised arithmetic gives rise to a revised algebra wherein any solvable equation can be solved exclusively within the real number system.**

**The significance of this conflicting methodology is that an equation such as " n x n = –1 " may be solved in either of two ways-**

**A. By creating the unit imaginary number that exemplifies the conventional system.**

**<u>OR</u>**

**B. By appropriately revising the rules of multiplication that exemplifies the revised system.**

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**Each man-made approach is equally arbitrary and therefore, equally justifiable.**

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**Under formalism, one of the two most widely accepted foundations for modern mathematics, any arbitrary set of basic assumptions or axioms that are self-consistent and thorough in describing mathematical reality is a legitimate model. However, only the most concise, simplest model is suitable as a general standard. Conventional algebra is universally accepted and used because it is agreed upon by experts as being such a model. Notwithstanding, the main thrust of this work is its contention that the revised algebra (and larger system) presented within is an even better model. Unfortunately, there are presently few experts, esp. those select, extremely few with the power to change worldwide mathematical standards, who are even aware of this work. [At least, not yet.]**

**In revised algebra, the imaginary unit and hence, the complex number system is completely unnecessary and useless. So, it is never incorporated to begin with since the revised real number system is omnipotent.**

**When it comes to choosing a hardline or softline position for the advocacy of either revised arithmetic or conventional arithmetic …**

**The softline position would be to state that both conventional arithmetic and revised arithmetic are as correct as they are relatively-consistent … despite whichever you prefer.**

**To state that both:**

> **" –1 x –1 = +1 " is correct according to conventional multiplication**
>
> **<u>and</u>**
>
> **" –1 x –1 = –1 " is correct according to revised multiplication**

**- is undoubtedly true since both systems of arithmetic are provably self-consistent.**

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**However, this statement diplomatically sidesteps being decisive about the obvious, critically-important issue as to whether conventional arithmetic or revised arithmetic is ultimately incorrect since they yield contradictory results. When all things are considered, it is possible to conclusively determine which is incorrect. The non-judgmental relativism inherent to the softline position becomes indefensible <u>if</u> either conventional arithmetic or revised arithmetic can be proven to be markedly superior to the other. In actuality, this is the case.**

**Most mathematicians I have rationally and tolerantly presented this alternative number theory to obviously hold a hardline position advocating conventional arithmetic. They have acted like an angry schoolmaster dealing with a bad student and said things to me such as,**

**“ –1 x –1 = –1 is dead wrong.”.**

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**Contrary to their intentions, I have been impressed only by their ignorance of the main purpose for the existence of MSC 03C62 (“models of arithmetic & set theory”).**

**Upon reflection, I have oddly decided to follow the less-than-inspiring example set by most ignorant mathematicians but only as far as to also settle upon a hardline position … advocating revised arithmetic, instead. So, I can act like an angry schoolmaster dealing with a bad student, too and say things to them such as,**

**“ –1 x –1 = +1 is dead wrong.”.**

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**For your consideration, I offer a complete, alternative number theory running appr. 250 pages which is chock full of rigorous, mathematical findings to support my hardline position. No non-trivial arguments for the contrary position- to prove and demonstrate how the conventional system is superior to the revised system in any way- are even possible.**

**Although I am an educated person, I am not a professional mathematician. Notwithstanding, I expect <u>any</u> person with the audacity to proudly call himself/herself a “mathematician” to at least, understand in theory how to perform simple arithmetic correctly (even though their jobs never require it) as thoroughly explained within this work. This is not an unfair or undue expectation on my part any more than, for example, expecting an intelligent child in his/her first year of elementary school to learn how to count.**

**In any case, I honestly predict that such an uneducated “mathematician” who does simple multiplication dead wrong yet naively and confidently thinks it is right will inevitably be remembered historically as a total disgrace to his/her profession as well as (to put it bluntly) a dumbass to the shocking extreme. Their place in history will be no better than, for example, astronomers in the 17$^{th}$ century who were familiar with the heliocentric theory yet deadset against it because they believed (some as religious fanatics) instead in the geocentric theory.**

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**In modern times, career mathematicians are controlled, intimidated and silenced by their fear of being labeled a "crank" and discredited by their colleagues <u>if</u> they dare to openly hold <u>any</u> radical ideas. The overall effect is that <u>all</u> established mathematical standards, even those that are dubious, must be uncritically worshipped or else, a person's career can be harmed or destroyed. Hence, it is no accident that the only people who have the prerogative to dare to point-out any possible mistakes in mathematical standards without risking reprisals are outside academia (such as myself).**

**The logical justifications and perceived necessities behind this cruel, severe treatment runs something like … despite their years of study, passing many classes and earning a degree or two, somehow their higher education failed to work on their innately-irrational minds and of course, standards of quality must be vigilantly protected. Admittedly, this actually happens to a small minority of people who have earned advanced degrees.**

**Notwithstanding, the main problem I find myself dealing with most commonly is essentially that vision, imagination, initiative, bravery and honesty are rare qualities to find at all amongst professionals within the natural sciences- perhaps because they are not taught or valued in the conniving, competitive environment of formal education. In fact, these defining qualities of individual intellect (as well as integrity and pride) are somewhat discouraged throughout modern academia.**

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**minimal completeness and maximal applicability**

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**Unless one blindly assumes the conventional system to have perfect, irreducible superstructure or "minimal completeness", then it is not necessarily impossible for a superior system to exist although it may thusfar be undiscovered or unused.**

**Unless one blindly assumes the conventional system to be perfect in the sense of having unrivalled ability to solve legitimate problems (algebraic and beyond) or "maximal applicability", then it is not necessarily impossible for a superior system to exist although it may thusfar be undiscovered or unused.**

**Unfortunately, for either ideal condition to actually exist within the conventional system would be a miracle since it has evolved and built-up over the centuries gradually, without following <u>any</u> overall, holistic design or long-term plan, always as an improvised, emergency response to the latest in a long series of utilitarian demands, into its present, asymmetrical superstructure in a piecemeal manner analogous to the spontaneous growth of spoken languages.**

**In summary, by following a pattern of development that was deficient in intelligent design and haphazard on every critically-important point, it was most likely doomed in every way to mature into something far from ideal which, not surprisingly, actually occurred with its present condition as an atrocious mess.**

**[This is an objective, factual assessment.]**

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**re: minimal completeness**

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**One capability of a superior system of arithmetic is enabling the solution of comparable, solvable algebraic equations from the conventional system in a more concise, simpler and structurally-symmetrical form. By reducing the number of required binary operations <u>or</u> number systems, two methods to definitively improve conciseness of form are identified.**

**In conventional arithmetic, there are six conventional binary operations existing as three pairs of inverses:**

- **addition (conventional) & (conventional) subtraction**
- **conventional multiplication & (conventional) division**
- **conventional involution & (conventional) evolution**

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**In revised arithmetic, there are only three revised binary operations:**

- **addition (conventional)**
- **revised multiplication**
- **revised involution**

**[Note: Since addition is a conventional binary operation regardless, only two of out the three so-called "revised binary operations" are literally revised.]**

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**minimal completeness**
**comparison #1**
**binary operations**

**revised arithmetic: 3**
**conventional arithmetic: 6**

**Revised arithmetic requires 1/2 as many binary operations.**

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**The revised binary operations are at least as capable as the conventional binary operations in arithmetical computation and serving in an algebraic framework (and other, higher branches of mathematics).**

**In conventional arithmetic, the real number system does not have closure under conventional involution and (conventional) evolution thereby creating complex numbers (and so forth to infinity).**

**In revised arithmetic, the real number system has closure under all revised binary operations.**

**Conventional algebra can solve <u>most</u> solvable equations within the complex number system, the fourth number system. However, an infinite number of hypercomplex number systems, creatable via the Cayley-Dickson construction, will ultimately be needed, in theory, to enable conventional algebra to solve <u>all</u> solvable equations.**

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**minimal completeness**
**comparison #2**
**number systems**

**revised algebra: 3**
**conventional algebra: infinity**

**Revised algebra requires an infinite fraction fewer number systems.**

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**Revised algebra can solve <u>all</u> solvable equations exclusively within the real number system, the third number system.**

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**minimal completeness**
**total comparison (#1 & #2)**

**Revised arithmetic and revised algebra require an infinite fraction fewer resources by measure in binary operations and number systems.**

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**These two vital comparisons necessitate that it is erroneous to attribute "minimal completeness" to conventional arithmetic and conventional algebra when revised arithmetic and revised algebra requires <u>an infinitely small fraction as many</u> binary operations <u>or</u> number systems to function effectively. Furthermore, they support a strong case for revised arithmetic and revised algebra having general superiority.**

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**re:  maximal applicability**

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**In conventional algebra, a binomial, linear equation to the fifth degree or higher is generally impossible to derive solutions for.**

**In revised algebra, a binomial, linear equation to the nth (<u>any</u>) degree is solvable since after revised cross-multiplication, it is always reducible to the original, first degree equation (which is solvable in every case).**

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**maximal applicability**
**comparison**
**solvable degrees of linear equations**

**revised algebra:  infinity**
**conventional algebra:  5**

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**Revised algebra has infinitely greater power to solve linear equations.**

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**This vital comparison necessitates that it is erroneous to attribute "maximal applicability" to conventional algebra when revised algebra has infinitely greater capability to solve linear equations.  Furthermore, it supports a strong case for revised algebra having general superiority.**

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**Conclusion-**

**In tandem, the comparisons of "minimal completeness" and "maximal applicability" wherein revised arithmetic and revised algebra are measurably, infinitely superior are severely damning to anyone who advocates and tries to justify the position that conventional arithmetic and conventional algebra should continue to be used.**

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**comparing numerical systems**

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**The revised numerical system of arithmetic, algebra, analytic geometry, analytic trigonometry and calculus presented and elaborated throughout this work is at least as internally, mathematically consistent as its counterparts under the conventional numerical system. Theoretically, any number of numerically and axiomatically self-consistent, invented systems can be created just as the modern, conventional system was created in the past. In fact, there are presently many models of arithmetic (MSC 03C62) available in the literature.**

**With adequate mathematical material provided, the revised system as presented can be verified as numerically and axiomatically self-consistent. Presumably, the material at hand also enables one to generate contradictions or isolate any unrecognized inconsistencies- fatal, major or minor. Ultimately, unless a fatal or major flaw is uncovered, then an objective evaluation of the two systems that are isomorphic in a holistic sense by their comparative attributes is required for defensible academic practices.**

**An objective evaluation is not a straightforward task, even for a person thoroughly knowledgeable in the relevant analytical and axiomatic areas. In actuality, it is very difficult to consistently distinguish between foundations, properties, structures and functions which are absolutely vital to <u>any</u> legitimate, universal system of mathematics and those exclusively characteristic of the conventional system which are merely relative, tenuous ramifications.**

**An indicative example of an abstract fixation, typical to the conventional viewpoint, manifests as an inadequate logical comprehension of this theory.**

**Under the revised system, after the unit imaginary number, complex number system (and in theory, an infinite number of hypercomplex number systems) have been precluded from existence at the level of revised arithmetic (specifically, in revised multiplication), it is impossible and unnecessary for them to mysteriously reappear in any way, either explicitly or implicitly, within legitimate problems in revised analytic trigonometry or revised calculus. In fact, <u>every</u> legitimate problem, interaction or phenomenon is now expressible within the revised real number system exclusively.**

**Various problems posed from the viewpoint of conventional mathematics may or may not have any theoretical existence, applicability or isomorphic solutions in revised mathematics. Nonetheless, the loss of those conventional problems (and their solutions) provably having absolutely no importance to revised mathematics would therefore also have absolutely no importance to modern mathematics based upon revised mathematics.**

**Ultimately, numerous value judgments are implicitly involved at foundational levels within each branch of mathematics. Guidelines to correctly make such determinations are not known in some cases. Nonetheless, all of this is prerequisite to being able to make an incisive, objective evaluation across the various topics encountered by this project. This illustrates the problematical nature of comparing various models of arithmetic in search of the best standard. After all, I think I can safely surmise that few mathematicians (who prefer calculable and provable problems) really want to address and deal with open, complicated, messy, quasi-philosophical matters.**

**By comparative criteria including (as well as going far beyond) those mentioned, a tentative determination as to which is probably a superior system can be made within a reasonable time. If said results are promising, then this could be followed-up by a more thorough, detailed and critical investigation of an abstract mathematical, computational, axiomatic, conceptual and foundational nature.**

**<u>Any</u> exact science (mathematics, most of all) is required to be correct, accurate and concise to the greatest extent possible. Therefore, if this theory is valid, substantial revision throughout arithmetic, algebra, analytic geometry, analytic trigonometry and calculus will be necessitated. The direct result would be a revised numerical system which is different computationally, markedly simpler, perfectly symmetrical and more applicable. Hence, the importance in making a definitive determination as to which is a better system vastly outweighs its required difficulty and commitment of resources. Few mathematicians, at any given time, are otherwise working on anything important, anyway.**

**Comparatively, no differences exist between the revised and conventional systems in pure geometry, plane trigonometry and vector algebra. There are only minor, formalistic adaptations involved with converting between the respective notations of the two systems.**

**Essentially, analytic geometry, analytic trigonometry and calculus are tools or techniques for abstract, exacting extrapolation from basic numerical and geometric truths. Defined along such lines, these higher branches of mathematics are wholly dependent upon the foundational branches (i.e., arithmetic and geometry) as the ultimate subjects of study.**

**In any event, analytic geometry and analytic trigonometry reflects the underlying differences in arithmetic and algebra between the two systems. Accordingly, two graphs of the same function or two functions defining the same graph are rarely or never identical between the revised and conventional systems of representation.**

**In turn, differential and integral calculus reflect certain underlying differences in analytic geometry and analytic trigonometry between the two systems with their unique, characteristic, contrasting function-graph relationships.**

**In summary, <u>all</u> branches of mathematics involving analytical/numerical systems, whether or not they also involve geometrical systems, are significantly affected having revised counterparts. Under consideration is most of pure mathematics and applied mathematics (at least, in their formal notation). Only exclusively geometrical systems are unaffected, remaining conventional in every case.**

**Although only a light survey of the various branches of higher mathematics, mathematical physics and engineering sciences has been undertaken, no legitimate branch, area or problem encountered thusfar presents a crisis or impasse to mathematical modeling under the revised numerical system. Moreover, a limiting mechanism to the otherwise theoretically-unlimited capability of representing an isomorphic, universal system has been shown (via this work) to be highly improbable. By definition, isomorphism between two universal systems is either fully-applicable or non-applicable. In other words, isomorphism between two universal systems either exists or does not exist.**

**All of the groundwork of this paper, with its interactive, mathematical self-consistency, was designed to prove the legitimacy and efficacy of the perfect symmetry number theory. Realistically, it is probably impossible to devise a non-isomorphic, fraudulent numerical system which can be methodically modeled and demonstrated through five branches of mathematics (arithmetic, algebra, analytic geometry, analytic trigonometry and calculus) with their intrinsic complexities and restrictions while retaining self-consistency, structures, perfect symmetry, <u>greater-than-maximal</u> applicability, <u>lesser-than-minimal</u> completeness (or conciseness of form). Therefore, it is far more likely that the revised system truly is isomorphic to the conventional system in its precise representation of universal, mathematical reality throughout calculus (and every natural science and mathematical science involving calculus, explicitly or implicitly).**

**Even if the revised (perfect symmetry) number theory is somehow not credited with general superiority to the conventional (broken symmetry) number theory, its definite, comparative advantages in many areas establish it as a productive, informative field of study and worldview in number theory, foundations and philosophy of math. In such a case, its recognized importance should be somewhat analogous to that of the non-Euclidean geometries in comparative geometry.**

**At least, with a bit of constructive creativity or imagination, it could be applied as a useful tool somehow, somewhere within the vast range and variety of mathematical studies and activities. In summary, the significance of this work to mathematics is virtually certain and guaranteed to those who have studied it and understood it.**

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**Alternatively …**

**Q- What would happen IF the revised (perfect symmetry) number theory were (justly) credited with general superiority to the conventional (broken symmetry) number theory?**

**What <u>would</u> become of the vast mathematical literature based upon conventional number theory?**

**A- What <u>should</u> become of the vast mathematical literature based upon conventional number theory.**

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**So, how can we correctly define "should" in this case? Realistically, I can only assess the error-ridden condition of it as totally hopeless.**

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**Accordingly …**

1. **<u>All</u> pure mathematics based upon conventional number theory could (and should) be safely discarded with prejudice immediately. Since nothing of value is known or probable to exist there, no rational purpose exists for anyone to try to salvage anything there. Over time, pure mathematics hopefully consisting of select, quality works with possible future importance would be rebuilt on an error-free foundation of revised number theory.**

2. **<u>All</u> applied mathematics based upon conventional number theory should be kept only until they can be replaced as soon as possible one-work-at-a-time by applied mathematics based upon revised number theory. Of course, this vast enterprise would require a lot of hard work (for a change) from a lot of applied mathematicians. After being successfully replaced by error-free counterparts, the original mathematical works could all be safely discarded with prejudice as well.**

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**The fact that many people for centuries have thought, worked and for the most part, wasted their entire careers creating this garbled mass of junk (with minimal value) called the worldwide mathematical literature is tragic and unfortunate. Hopefully, mathematical academia can and will learn from its huge, costly, unsalvageable mistakes and numerous, long-standing "bad science" practices that blatantly violated the scientific method and at least, do a better job next time. Notwithstanding, there is no justification at all, by scientific and academic standards, for sentimental attachment to mathematical works once they have become useless, obsolete and provably wrong or erred.**

**An alternative number theory can overwhelm the patience and adaptability of many mathematicians. Nonetheless, since the heart of the perfect symmetry number theory lies within arithmetic, it is accessible, readily-provable and can clearly be envisioned without excessive dependence upon mindbending, abstract mathematics.**

**Three demo programs for revised arithmetic (and areas foundational to it) provide immediate, clear feedback and confirmation when one is on track in understanding this alternative yet superior number theory. The effort invested reveals very interesting theoretics as well as issues of importance to mathematics as a whole.**

**By the way, the indisputable fact that demo programs for revised arithmetic exist and work perfectly instead of generate inconsistent, random and/or useless answers that cannot be rationally worked back to their starting place is <u>evidence and working proof of the self-consistency of revised arithmetic</u>.**

**Rest assured, I understand and accept that anyone has the right to tell me,**

> **"You are making a strong assertion.**
> **So, the burden of proof lies entirely upon you.".**

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**This demand is reasonable and consistent with the scientific method. After all, skepticism is the nature of good science and its standards of quality must be protected. In the course of this work, I have compliantly attempted (successfully, in my studied opinion) to fulfill this demand in many ways.**

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**For an analogy with a historic twist …**

**I would expect that if I told a physicist over a century ago that I knew how to build the first working radio, even if my explanation made sense in principle, he/she would naturally be skeptical that I could really turn the dream into reality. However, I would <u>not</u> expect to still be treated like a dumbass or nut <u>after</u> I turned the device on and let him/her hear it broadcast.**

**I hate to inconvenience anyone by putting them into the uncomfortable position where I am rightfully demanding that they actually wake-up, act like responsible mathematicians and do some work (as I have) of importance via examining my proof and verifying it but I have laid it all out for you neatly "on a silver platter" … and I am still waiting.**

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**Apparently, "the system" has no provisions in place (although it should) for a rare (presumably) yet undeniably-possible, human error situation that ideally, should never occur yet realistically, almost certainly has occurred and will occur in numerous unknown places within the vast body of mathematical literature where an error in mathematics by one mathematician is <u>not</u> caught by peer review (if any) in the era (in some cases, ancient) when it is originally made. So, <u>the error is admitted into mathematics and becomes a standard</u>. In subsequent generations, <u>all</u> of the standards of mathematics are vigilantly protected … including the error, unfortunately.**

> **Q- What can be done to purge an error from the standards of mathematics and repair or correct the standards of mathematics accordingly?**

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**"Nothing" is not even close to an acceptable answer.**

**If the modern-day reality is that "the system" absolutely does not ever function that way, then "the system" needs to arbitrarily reform itself to function that way- only in highly-unusual situations that justify it. There is no adequate, ethical or academic, excuse for the mindless perpetuation of "dereliction of duty" in the natural sciences through an unknown number of future generations.**

**Please try to understand it from my point of view?**

**Since I am convinced that a serious error (obvious, stupid and directly causing bad consequences) has been admitted into, never detected and never removed from basic arithmetic (after appr. 14 centuries), at relatively the lowest level of abstraction, it seems overwhelmingly probable that undetected errors (some serious) must also exist in moderate numbers at the medium level of abstraction and must also exist in great numbers at the highest level of abstraction. Thus, I am incapable of trusting any reassurances to the contrary from any "experts in mathematics" who routinely use the same type of circular logic to self-justify their positions as people who have thrown me out of their offices for trying to explain how conventional multiplication is flawed.**

**When you observe that for many years, virtually all mathematics journals have been filled with works at the highest level of abstraction, intentionally and somewhat unnecessarily for prestige, then you may wisely reconsider my assessment that the worldwide mathematical literature is "a garbled mass of junk (with minimal value)" as <u>not</u> being too harsh after all. I would dare bet that virtually all mathematical works at and above a presently undefined (perhaps, moderate) level of abstraction, are so riddled with errors and/or heavily, foundationally based upon errors, they will eventually be discovered to be worthless for all practical purposes. Of course, this will be true to a much greater extent for pure mathematics than applied mathematics.**

**an unnatural history**

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**It is pointlessly masochistic that we (i.e., all of humanity) are still unnecessarily choosing to suffer for a fundamental mistake made in the development of mathematics during times of antiquity, reportedly by Brahmagupta of India (circa 628). This respected and productive [most of the time] mathematician incompetently devised the peculiar, "self-trapping" method of multiplying positive and negative factors that became the worldwide standard for multiplication.**

**Of course, when one further considers the ramifications such that conventional multiplication and thus, conventional involution give rise to a conventional algebra in which there does not exist the capability to solve some imperatively-solvable, simple equations within the real number system, inadequacies and crises compound. As a direct result, the imaginary unit and complex number system had to be immediately invented sheerly to enable their solution.**

**When confronted with such a formidable shortcoming in the capabilities of conventional multiplication within conventional algebra, it is surprising that instead of re-evaluating conventional multiplication in search of some basic error or limitation that could have easily been found (to make the correction identical to that presented within this paper), a dogmatic position was stubbornly maintained wherein it was assumed in absolute terms that no error in conventional multiplication could have possibly been made or thus, could currently exist within it.**

**Upon such arrogant logic, it was indisputably further assumed by the mathematical establishment of centuries ago that there inexplicably existed ample justification for the arbitrary creation of one new number system, the complex number system. [Delays and controversies, notwithstanding.] Ironically, the creation of the complex number system was absolutely necessary to enable conventional algebra to work-given the restraining, flawed assumption that conventional arithmetic with the real number system was inerrant and structurally-simplified. Of course, the huge ramification completely missed and not predicted at all during the era of its invention a few centuries ago was that ample justification for the arbitrary creation of an infinite number of hypercomplex number systems had also been assumed which is cumbersome and problematic.**

**With a holistic overview now afforded to us by historical developments spanning appr. 14 centuries, we can now easily see the obvious that the mathematical establishment of antiquity essentially painted itself into a corner (through abysmal lack of foresight) and then later, cheated to escape the trap (that was its own fault for creating).**

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**For an appropriate analogy …**

**It is not at all surprising to witness a novice at Chess playing into direct, catastrophic traps due to his/her inability to think clearly and comprehensively only 1-2 moves ahead in complicated situations. What is surprising is for such a gross incompetent to insist upon arrogantly calling himself/herself (and being called) an official "master of the game", dishonestly or close-mindedly refuse to admit to making any mistake (despite the bad outcome that is painfully evident for anyone to see) and rant at anyone (esp. someone who is <u>not</u> also an official "master of the game") who dares to correctly point-out his/her error.**

**If this ridiculous folly had not caused several serious, lasting dilemmas for mathematics (and in turn, most natural sciences), it would be humorous. Instead, it is such an overwhelming testament to and absolute proof of the astonishing levels of stupidity and/or ignorance still prevalent within the minds of <u>virtually all</u> 21st century mathematicians that, after it is inevitably straightened-out, educated people from future centuries will certainly be contemptuous, dismayed or puzzled. They will probably also have considerable difficulty seriously believing or accepting that such a travesty really could and did happen as well as coming to grips with how it could possibly happen.**

**They will surely be resolutely disrespectful and derisive toward the memory of those leaders and members of the mathematical establishment who actively, shamelessly fought against the correction of serious errors in basic arithmetic even after they had been pointed-out clearly, explicitly and exhaustively. The culpability of all individuals who are paid to advance science yet cynically, secretly choose to be enemies of science, knowing they can get away with it, just to complacently avoid the disruption that progress/change entails, is much too high to be forgivable.**

**When individuals are, by strict policy, rewarded greatly for compliance and punished severely for defiance by an educational institution, important, large and disruptive reforms <u>never</u> occur.**

**What we are witnessing is not merely an innocent (although serious) theoretical mistake in the historic development of mathematics but instead, a continuing compounding of a root, serious, theoretical mistake (with devastating consequences to the intelligibility and symmetry of the mathematical literature) and its intentional, widespread cover-up spanning at least a few centuries- <u>always to prevent any major disruption in the basic textbooks and mathematical literature for the benefit of those experts currently in power</u>. Of course, this could not have been accomplished without the arrogant, corrupt disregard and defiance of any/all evidence and quality ideas to the contrary known at the time by those who were well-informed (and there have always been some).**

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**In centuries past, it was especially easy for the history of mathematics to be written by the corrupt victors of all disputes (similar to this one) who successfully disposed of virtually all evidence of dissent or at least, all evidence of dissent that was rational and intelligent, thereby leaving modern, objective historians of mathematics with little or nothing to justify the position of dissent. Fortunately, it has become more difficult for the status quo to keep secrets in the modern, internet age.**

**In case you are wondering …**

**No, I am not falling for an inviting paranoid or contemptuous fallacy, characteristic of many conspiracy theories, due to an unrealistic expectation that mathematical institutions and their leaders, esp. in centuries past, should have operated and thought perfectly. I do not ever expect perfection.**

**The topic at hand is not whether the first serious mistake to basic arithmetic, committed in ancient history, occurred accidentally. In fact, I have no reason to doubt that indeed it did occur accidentally. Rather, the topic at hand is why and how such a serious, fundamental mistake (painfully evident to anyone <u>with any sense</u> who has examined it as well as an impossible topic for any educated mathematician to have not been required to cover) neither has been nor is in the process of being corrected. After all, we live in a age where professional mathematicians have, by far, the greatest resources ever in history at their command- human, computer, technological, financial, etc.**

**Overall, this self-serving, corrupt pattern of behavior, consistently demonstrated by mathematical academia worldwide for many centuries, could not have caused any effect other than to slow and degrade human progress educationally, technologically and economically. Ironically and hypocritically, academic mathematicians brazenly and dishonestly take as much credit as they can for all human progress to date from people in other walks in life who typically are naïve about the unexpectedly-disgraceful history as well as present-day workings of mathematical institutions.**

**Q- How can someone who is highly-educated, well-paid, well-treated and respected be foolish or corrupt to such an extreme that he/she is willfully an agent for stagnation who uses all of his/her bureaucratic power to defeat all "disruptive" reforms and ideas that would be highly beneficial?**

**Q- How can someone who owes their highly-privileged existence in society to the greatest ideal in science (mathematics) care so little about it and be willing to do so little for it that they allow their overall societal effect to definitely be as an enemy of progress?**

**We can be relatively sure that various leaders and prominent individuals with the power to control or influence mathematical academia have been behaving very badly for centuries and having their way. [Not just historically but also presently.]**

**With respect to those mathematicians who knowingly allow serious, fundamental errors to persist in mathematics, the best analogy I can think of is to liken this corrupt behavior to that of bad, spoiled children who are drunk with power, throw temper tantrums at will or whim and <u>always</u> get away with it. Of course, nothing provokes a worse temper tantrum than whenever anyone dares to try to correct them over anything since they believe themselves to be the "smartest of the smart" who never make or perpetuate mistakes.**

**[Note that no fair consideration of the critical point this person made ever occurs.]**

**In a neverending way, they stubbornly refuse to maturely address, truthfully admit to and correct any of the consequences of their own bad behavior, mathematically or socially, yet the only feedback they ever experience is that they are given total, undeserved, unearned victories every time. Unfortunately, until/unless their power to behave as badly as they wish, anytime they wish, without any repercussions is completely taken away, no aspect of their behavior will ever improve at all and nothing constructive of non-trivial value will ever be accomplished throughout their entire adult lives. Yes, we still live in a world where mathematical academia is totally out-of-reach from any conceivable reforms from outside itself under the established power structure and its corrupt, egomaniac leaders know it quite well, with confidence.**

**The foundational errors in constructing conventional arithmetic are so extreme, it is literally inconceivable how they realistically could have been made any worse. With conventional multiplication involving positive and/or negative real numbers corrupted, one can only wonder if it is even possible for any sane person to have unintentionally corrupted the only simpler binary operation, conventional addition, involving positive and/or negative real numbers.**

**In fact, if they had somehow managed to mess-up conventional addition as well, the results would have grossly, measurably contradicted real-world experience to such an extreme that it would have been evident to intelligent laymen and turned "number theorists" into "numerologists" (as outcasts from society) in a likewise manner as incompetence-to-the-lunatic-extreme can turn "astronomers" into "astrologers" (as outcasts from society).**

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**Therefore, a reasonable textbook definition of a "dumbass" as being, "Someone who thinks something so extremely dumb, it could <u>not</u> have practically been exceeded." is evidently an appropriate, fair and unexaggerated way to describe a typical 21st century mathematician who believes in conventional multiplication as being correct and accurate. In dramatic contrast to their own assessment of their intelligence, knowledge and vision, typical, modern mathematicians <u>provably</u> cannot see mathematical reality clearly and correctly any further than what is physically-evident, direct experience … right in front of their faces. <u>Everything</u> past that has been distorted to such an extreme that it is only an incomprehensible blur to them.**

**Unfortunately, problems compound with each more abstract branch of mathematics that is successively built upon the unsound, asymmetrical foundation of conventional multiplication (within conventional arithmetic). This means that more problems exist for conventional algebra than conventional arithmetic and likewise, more problems exist for conventional analytic geometry, conventional analytic trigonometry and conventional calculus than conventional algebra.**

**An appropriate analogy for the naïve effort (currently underway and more active than ever before in history) to correctly build evermore-sophisticated analytic/numerical branches or specializations of mathematics ultimately based upon conventional arithmetic is …**

> **- trying to build more floors in a neverending way onto a skyscraper with a broken, lopsided, asymmetrical foundation.**

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**Ultimately, the effort is doomed to stagnation and failure- regardless of the amount of ingenuity applied to it. Each successively-higher floor becomes exponentially more unstable, complicated and difficult to build. A law of diminishing returns soon sets in with it eventually becoming impossible (or virtually so) to successfully build any more floors. This effect is already evident and commonly noticed (although misinterpreted) in modern mathematics where extreme efforts of abstraction are required in modern times to create anything new and only the researchers understand (or imagine they understand) their own work with such creations always being of trivial or unknown [translation: zero] importance.**

**It is past time for the old building to be condemned and destroyed in order to clear the construction site for a new building. The major mistakes made in the unsuccessful construction of the old skyscraper can only be used as abject lessons to be avoided and thereby instruct us in how to successfully build a much better, new skyscraper from the ground up.**

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**unclear foundations of math**

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**Mathematicians are in an ideal position to fully appreciate what is meant by the well-known "unreasonable effectiveness of mathematics to scientific endeavors" and "unreasonable ineffectiveness of philosophy to scientific endeavors". Specialists in foundations and/or philosophy of math sometimes over-estimate the importance of their work to those in other specialties. In fact, few mathematicians are typically concerned on a daily, working basis over logicism, formalism or any other philosophical position. Instead, their primary concern is that the mathematical enterprise as a whole always remains productive … as evident by the work they are doing at the moment.**

**Typically, they see this as insured by remaining open-minded, practical and busy; as potentially threatened by becoming overly-ideological, fanatically reductionistic or lazy. I do not know of a name for this "philosophical position" but it may be the one most mathematicians adhere to more strongly than any traditional, philosophical position they also favor or agree with (if any).**

**Although noone should dismiss the correctly-defined foundations of math as being unimportant, there are quite possibly a few serious problems and limitations with rigor in <u>all</u> present-day programs which attempt to do so. Moreover, there are unavoidably implicit value judgments of a quasi-philosophical nature involved that are subject to sizeable human error. Unfortunately, those who demand a clear demarcation between mathematical foundations and philosophies (to avoid dealing with disagreements, confusion and pseudo-scientific "works of science" inevitably created in a setting of philosophizing) desire the impossible. Total commitment to a mislaid foundation of math would eventually yield disastrous consequences.**
**In principle, taking such a dangerous gamble without being forced to would be unwise. [Note- We are not being forced to.]**

**Most mathematicians regard the theoretically-infinite universe of possible models of arithmetic (and math) as trivial compared to other vital areas of mathematics.**
**In every case except ONE, I actually agree.**

**A sharp distinction should be made between applicable and non-applicable models of arithmetic (and math). Applicable arithmetics (and maths) are defined herein as those which are provably, measurably compliant with physical reality, those which can be applied to our universe. For instance, all of the interactions of exclusively positive real numbers under the three binary operations <u>must</u> be defined by the familiar, conventional standard. To be otherwise in any imaginable way, they would then be measurably incorrect. Non-applicable arithmetics (and maths) are defined as those which are NOT provably, measurably compliant with physical reality and cannot be applied to our universe.**

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**Guesswork at the correct mathematical modeling of possibly non-existent, additional universes (which never can be confirmed, observed, studied) is NOT a scientific endeavor. Furthermore, it fails to meet the criteria of an intelligent, productive or even, rational endeavor. Therefore, all studies of non-applicable arithmetics (and maths) should be terminated.**

**By contrast, exploring applicable arithmetics (and maths) is potentially significant and productive since they can at least be compared meaningfully to the conventional model. Of the numerous ones I have studied, experimented with and/or invented, some have been a little better than the conventional model; some have been a little worse. Remarkably, I have only discovered <u>one</u> applicable arithmetic (and math) that is far better than the conventional model.**

**This alternative model has the distinction of being the only one that can be based upon numerical perfect symmetry. [Note- It also possesses geometrical perfect symmetry via its characteristic function-graph relationships.] It is the only model I have ever discovered (and probably, the only model theoretically possible) whose comparative merits outweigh its costs (i.e., temporary disruption) of <u>replacing</u> the current standard model by far.**

**Although I have done research, I did not discover this model pre-existing in the literature. Instead, I discovered and modeled it from scratch while attempting to invent an unconventional system whereby real numbers possessed ALL of the problem-solving capabilities complex numbers were normally required for under the conventional system. Unexpectedly, I easily succeeded.**

**[Note: This achievement is next-to-nothing for me to brag about. It would have been easy, even for an untrained amateur, to do equally well or a bit better. <u>Anyone</u> could have and should have done it many years earlier than I if they had merely cared enough to try.]**

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**For comparison:**

**In computer science, where there are many constant pressures or demands from the outside world that must be met, professional standards are measured and expressed primarily in terms of efficiency. If a new programming language were introduced which possessed appr. 5 times more complication than absolutely necessary (without any compensating advantages), it would quickly be doomed to extinction with prejudice as it was replaced in favor of a better programming language that could be developed using routine methods.**

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**In pure mathematics, where there are few pressures or demands from the outside world which must be met, professional standards are measured and expressed primarily in terms of abstract grasp of convention. The luxury of remaining mostly unfamiliar with the concept of efficiency is commonplace. [This is significantly moreso the case in modern times than it was in ancient times.] Unfortunately, this situation gives too much free reign for sentimentality, tradition, stagnation, discrimination, fastidiousness and unconditional protection of the status quo. Consequently, an established mathematical language that possesses at least 5 times more complication than absolutely necessary (without any compensating advantages) is NOT in any imminent danger of extinction, whatsoever.**

**Instead, its ideal-replacement, mathematical language has no secure future to date and as such, remains in danger of extinction inevitably unless-until a dramatic, fundamental improvement occurs in the situation. There remains hope since it is always possible that its merits will be recognized and appreciated anytime yet realistically, it is difficult to imagine exactly how or when progress can take place.**

**It is not my radical contention that it is impossible for an intelligent person to learn some important matters about mathematics using the established, over-complicated language, regardless.**

**I consider this analogy an appropriate description of my position:**

1. **A person with perfect eyesight is unnecessarily forced to wear eyeglasses with thick, strong lenses at all times to see the world.**

2. **With great effort and practice, this person eventually can see adequately well.**

3. **Still, this person would be able to see reality a lot more clearly and easily without the eyeglasses.**

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**The moral of the story is that freedom and empowerment can be available as the simplest, easiest option imaginable yet inexplicably, irrationally may not be chosen.**

**This project is mainly about practical, constructive mathematics that would provably be beneficial. Essentially, it is about "how to build a better mousetrap". To be sure, it is not about my philosophical ramblings or radicalism (if you perceive some of my points that way).**

**One need not be a formalist to find something of value within this work. Please exercise intelligent judgment, good taste and ethics in deciding what you value. Please take measures to promote, preserve and protect what you value.**

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**the non-absoluteness of mathematical proofs**

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**Ultimately, mathematical proofs for the foundations of a numerical system establish nothing more than internal, mathematical consistency. In fact, many of the statements within mathematical proofs for the foundations of revised arithmetic are characteristic of and true for revised arithmetic exclusively. This situation is essentially self-justified or circular logic. Nevertheless, the converse is also true for conventional arithmetic, its proofs having the same limitations.**

**The principle to be cognizant of in the comparison of the two distinct, numerical systems at hand is their incompatibility although conversion/translation between them is possible.**

**Paradoxically:**

> **By the arithmetic of the conventional system, the conventional system can be proven as valid <u>and</u> the revised system can be invalidated.**
>
> **By the arithmetic of the revised system, the revised system can be proven as valid <u>and</u> the conventional system can be invalidated.**

**Consequently, unerred mathematical proofs for either system can only be tentatively accepted to a limited extent. All mathematical proofs against the basic validity of either system must be invalid or erred somehow because they are not, in of themselves, capable of the needed scope and value judgments to decide the comparative, holistic advantages and disadvantages of two distinct, isomorphic systems.**

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**an unconventional, direct approach**

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**The conventional methods established for communicating works in models of arithmetic can be useful yet they are abstractly based entirely and expressed entirely in terms of conventional mathematics (of course) that can entail drawbacks.**

**Although revised mathematics can be converted/translated into conventional mathematics and vice versa, expressions from revised notation constructed using/within conventional notation are compounded, messy and complicated, even to equations which are relatively simple, given under either revised notation or conventional notation. In any case, a more concise, symmetrical, revised, unconventional model, presented in such a convoluted form, would NOT likely be recognizable as such nor understood as easily as possible IF a more-practical approach existed.**

**Departures from the conventional notation, if constructible/possible, are allowed whenever unavoidable or advantageous in models of arithmetic. The exposition of this theory benefits greatly from such a departure. "Conventional unconventional arithmetic" is not an oxymoron by accident. Moreover, conventional presentation is a serious yet unnecessary roadblock to some unique, conceivable, useful presentations in or relevant to models of arithmetic.**

**The approach used throughout this paper is to teach revised mathematics in a manner parallel to how we all learned conventional mathematics as school children <u>and</u> parallel to the historic development of conventional mathematics. It takes the form of a fundamental, educational exposition of a general theory of mathematics-NOT as a highly-abstract, specialized journalistic article. The focus is upon educational clarity to methodically build-up necessary foundational concepts, similar to as you would find in the structure of a textbook about fundamentals.**

**Accordingly, the fundamentals of arithmetic, algebra, analytic geometry, analytic trigonometry, calculus under the revised system are presented in their own directly-understandable terms, concepts and axioms with sparingly few presumptions upon the reader to possess any advanced or much previous education in mathematics.**

**With all necessary explanations, examples and explicit details included along the way, any mathematician or educated layman should be able to successfully learn the basics of the perfect symmetry number theory with concreteness and certainty. No more time and effort than absolutely necessary are imposed upon. Nonetheless, it would be terribly unrealistic, even for a learned person, to expect to master an alternative number theory in a day.**

**Although this is an unconventional approach, it is directly accessible to nearly anyone (even to a person with little formal education in mathematics), familiar to our past mathematics education (as children and teenagers), familiar to the history of the development of mathematics and comparatively-advantageous to a confusing, extremely complicated, mixed approach of using conventional notation conversions/translations to build this unconventional model.**

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**representations with rectangular coordinate systems**

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**<u>Any</u> arbitrarily-enumerated x and y axes under a given binary operation will yield values (c) outside themselves across the x y axes plane which are relatively self-consistent within the rectangular coordinate system. Furthermore, there should be no discrepancies between the values along the x and y axes (a & b) <u>and</u> the values along an identical line representing sums (c), products (c) or powers (c) for that indicated interaction of x axis and y axis values (a & b) under the corresponding binary operation. In other words, every single point along these two identical lines must match exactly at one unique value.**
**To otherwise have an irreconcilable situation in which two different values must exist for a single point is a mathematical self-contradiction which is unallowable.**

**This crisis occurs in most conventional binary operations- namely, subtraction (conventional), conventional multiplication, division (conventional), conventional involution and evolution (conventional). By contrast, no such crisis ever occurs in any revised binary operations because every point along the x axis and y or y± axis/axes are in their only, correct geometrical and numerical inter-relations within the rectangular coordinate system whereby their original values (a & b or b±) are restated (c) by the identity elements of the appropriate revised binary operation.**

**With every conventional binary operation, the rectangular coordinate system is used in the same manner. The x axis and y axis are ordinary real number lines which intersect perpendicularly forming an origin at (0, 0).**

**In subtraction (conventional):**

**Every point on both identical lines, the y axis (b) and the differences line (c) where the x axis (a) equals "zero", have corresponding values which are comparatively opposite for every case except zero.**

**In conventional multiplication:**

**Every point on both identical lines, the x axis (a) and the conventional products line (c) where the y axis (b) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "a = 0" .**

**Every point on both identical lines, the y axis (b) and the conventional products line (c) where the x axis (a) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "b = 0" .**

**In division (conventional):**

**Every point on both identical lines, the x axis (a) and the quotients line (c) where the y axis (b) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "a = 0" .**

**Every point on both identical lines, the y axis (b) and the quotients line (c) where the x axis (a) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "b = 0" .**

**In conventional involution:**

**Every point on both identical lines, the x axis (a) and the conventional powers line (c) where the y axis (b) equals "zero", have corresponding values of the set of real numbers and "+1", respectively. Clearly, these values are different for every case except where "a = +1" .**

**Every point on both identical lines, the y axis (b) and the conventional powers line (c) where the x axis (a) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "b = 0" .**

**In evolution (conventional):**

**Every point on both identical lines, the x axis (a) and the roots line (c) where the y axis (b) equals "zero", have corresponding values of the set of real numbers and "+1", respectively. Clearly, these values are different for every case except where "a = +1" .**

**Every point on both identical lines, the y axis (b) and the roots line (c) where the x axis (a) equals "zero", have corresponding values of the set of real numbers and zero, respectively. Clearly, these values are different for every case except where "b = 0" .**

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**in search of intelligent life**

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**It is highly unlikely that other technological civilizations within our part of the galaxy are only appr. 100 years advanced past the crude invention of radio (for example). After all, on an astronomical scale, 100 years is barely "a tick of the clock" (so to speak). Probabilistically, where technological civilizations exist, those 10,000-100,000 years advanced (at least) would be much more commonplace. Relatively speaking, it is difficult to imagine a technological civilization more primitive than our own that meets the minimum qualification as a technological civilization.**

**A bored adventurer from a highly-advanced alien civilization (named Mork) decides to risk making contact with the dangerous aborigines of Earth and if he survives, make a report. So, he studies technical English, lands his spacecraft on the White House lawn and gets out. After waiting a while, he is greeted by the director of the National Science Foundation (named Mindy).**

**Based upon the primitive examples of technology (electronic and mechanical) that are visible around him, Mork is optimistic that some knowledge and mastery of basic, applied mathematics must exist.**

**During his brief visit, his sole mission is to assess the level and quality of mathematical understanding that the human race possesses in order to expediently grade their intelligence (since it is the most important, predictive indicator of their likely rate of future, technological progress).**

---

**The conversation begins …**

---

**Mork: Hello.**
**I come in peace.**
**Please don't kill me?**

**What is <u>the</u> square root of +1?**

**Mindy: Hello.**
**Thank you for not destroying our planet.**
**Welcome to Earth!**

**There are actually <u>two</u> square roots of +1 …**
**+1 & –1.**

**[Mork momentarily lapses into a mild state of shock and disbelief.**
**Then, Mork regains his composure.]**

**Mork: Really? I understand how you got the answer +1 and I agree that answer is correct but how did you also get the answer –1?**

**Mindy: Well …**

$$-1 \times -1 = +1$$

**<u>and</u>**

$$+1 \times +1 = +1$$

---

**Mork: Really? Wait. What can you actually do with a square root of a positive number that is negative, the –1 answer, in applied mathematics?**

**Mindy: We just arbitrarily throw away the –1 answer since I agree, it is obviously useless. Besides, we have a square root of a positive number that is positive, the +1 answer, which can be used in applied mathematics.**

**Mork: Yes but how can you justify arbitrarily throwing away one of your two answers? Mathematics is supposed to be the most serious science.**

---

**Mindy: I don't know exactly how. Frankly, I don't even care. Ask one of our expert mathematicians the details. Of course, this person will not care either but will know the reason. Generally, noone really cares at all although we all must lie to the public and promise we care a lot.**

**The point is we are such cleverly-manipulative educators and scientists, we can always invent any needed flimsy, theoretical rationalization to arbitrarily justify anything we really want to do or really want <u>not</u> to do. It completely satisfies the lax standards we half-ass apply to ourselves and our students don't dare argue.**

**After all, we are the only people on the planet whose opinions matter at all. We are the elites and the official experts. We are the academic authorities who have power in this manner worldwide in an actual totalitarian sense, even into and including democratic nations. We tell all of the world's people what the standards are. We tell all of the world's people what is correct and incorrect. Noone dares to tell us, instead, what the standards should be or what is correct and incorrect, regardless of the strength of their mathematical arguments … precariously and humorously assuming that any of us ever waste any of our time reading such insulting things.**

**Mork: Are those your highest ideals of good science, good education, fair academic practices and ethics?**

**Mindy: Yes.**

**Mork: I understand but how does your worldwide, totalitarian, elitist-ruled, no-dissent system of mathematics ever self-correct when mistakes are made in establishing standards?**

**Mindy: It doesn't ever self-correct but the beautiful thing about our system is that since it works perfectly, it doesn't ever need to self-correct. So, there isn't any problem at all.**

**You see, all of us throughout the entire history of mathematics have always been such miraculous super-genii that we have never made any mistakes at all in thousands of years of arbitrarily creating new branches of mathematics, accommodating new works and establishing standards.**

**Mork: Really? How do you know that for certain?**

---

**Mindy: Just examine the situation logically. Then, the right conclusion is inescapable.**

**If we had ever made any mistakes in establishing standards, then all of the many miraculous super-genii alive today within our worldwide army of mathematicians would have surely detected them, pointed them out to others and corrected them. Therefore, the realistic odds of any mistakes whatsoever currently existing anywhere within modern mathematics are zilch.**

**Trust us!**

**Mork: It is difficult for me to trust you when you contradict yourself logically. You previously stated the system does not have the power to self-correct. Then, you stated that if any mistakes were detected by professional mathematicians, they could and would be corrected. How?**

**Mindy: Hey, alien smart ass! If a majority of us elites detected a mistake and wanted to correct it, we have the power to do so even if it is unprecedented.**

**Mork: Fine but what if only a minority of the elites detected a mistake and wanted to correct it with the majority of the elites being against doing so, indifferent or uninformed about the matter.**

**Mindy: Then nothing would happen.**

**Mork: That sounds like a serious problem to me.**

**Mindy: No. That isn't a problem at all. As I said before, I have faith that a majority of our miraculous super-genii would detect any real mistake and vigilantly correct it. Hypothetically, I guess it is possible a minority of our miraculous super-genii could be mistaken in thinking there was a mistake. It doesn't matter, though, since that has never actually happened.**

**Mork: That isn't a realistic expectation. Mathematicians are separated into too many specializations for a majority to be aware of any specific mistake.**

**Mindy: Like I said, we don't make any mistakes and never have. So, even <u>if</u> your point is logical, it is worthless in practice.**

**Mork: Every person and institution in the galaxy presently makes mistakes.**

**Mindy: Except us!**

**Mork: Really? [He resumes his point despite the loud interruption.]**
**This is even moreso true in the ancient past and under primitive conditions like existed until very recent centuries on Earth. Therefore, having and allowing no administrative mechanisms to exist for the correction of mistakes can be disastrous to the future development of an enterprise such as mathematics.**

**Mindy: Not for us!**

**Mork: Really? Then, what is the square root of –1?**

**Mindy: In the set of real numbers, there is no square root of –1.**

**Mork: There must be a square root of –1!**

**Mindy: Give me time to explain! There is a square root of –1. It just isn't a real number, in this case.**

**Mork: What else can there possibly be beyond real numbers?**

**Mindy: Interesting! Obviously, our mathematicians know a Hell of a lot more than your alien mathematicians. Hey! That is a very nice spaceship you are flying but your alien mathematicians are all dumbasses compared to ours.**

**There is the unit imaginary number, the complex number system and theoretically, an infinite number of hypercomplex number systems.**

**Mork: Really? All we have ever needed in 50,000 years of scientific history to do arithmetic and algebra is real numbers. Please give a straight answer? What number multiplied by itself equals –1?**

**Mindy: The unit imaginary number "i".**

$$i \times i = -1$$

**Mork: What is "i", though? Define it.**

**Mindy: I just gave you the only definition of "i" that exists. It simply is the number multiplied by itself that somehow, inexplicably equals –1.**
**Of course, it cannot be a real number since I have already explained how multiplication with exclusively positive or negative factors works.**
**It is just "the unit imaginary number".**

**Mork: "Imaginary" as in not real at all?**

**Mindy: No. Now, don't be sarcastic. The unit imaginary number is serious stuff- mathematical reality to us. "Imaginary" is just the arbitrary name we gave it that should not be taken literally.**

**Mork: Every number that exists except zero must be either positive or negative.**

**Mindy: Again, that just proves how astonishingly little you aliens know about arithmetic. How can you be so dumb and have any idea how to fly your spaceship?**

**No. The unit imaginary number is neither positive nor negative.**
**It is magically beyond positive or negative. Ask one of our expert mathematicians the details.**

**Mork: Well, did you arbitrarily invent more than two signs of numbers- positive and negative- to somehow solve for the square root of –1 or not?**

**Mindy: Of course not. We all know and agree there are only two signs of numbers- positive and negative. It is just that classifying numbers as either positive or negative only goes as far as the real numbers.**

**Mork: Well, you are approximately right but for all of the wrong reasons.**

**Mindy: What?**

**Mork: Nevermind. Then are you certain that your so-called "unit imaginary number" isn't actually, perhaps in an unrecognized manner, just –1 (as a negative real number) operating under different rules of multiplication like our alien civilization uses whereby two negative factors multiplied equal a negative product?**

**Mindy: Yes. It definitely is not that.**

**Mork: Then what is it? How do you know for sure? After all, it is isomorphic to and indistinguishable in its product from what I described.**

**Mindy: I don't know exactly what it is. I just definitely know what it is not. Don't argue with me! I have already ascertained that Earth mathematicians are much smarter and more knowledgeable in numerous ways than any of you ordinary aliens can possibly be at math.**

---

**Mork: You did not convince me. You did not even make a mathematical or logical argument. Do not tell me what I cannot do. I am going to argue with you, anyway, because you need to understand:**

**If you state …**

$$n \times n \neq -1$$

**- then you should not just substitute the letter "i" for "n" to mysteriously solve the equation.**

$$i \times i = -1$$

**If you disregard my advice and do it anyway, then essentially all you have accomplished is to cheat to expediently and sloppily solve the problem one step further back than necessary with the unit imaginary number after and due to failing to solve the problem where you should within the set of real numbers. This will overcomplicate and cripple your entire system of mathematics dreadfully.**

**Mindy: I wish you had a clue what you are talking about. Our system of mathematics works fine. Frankly, I can clearly see your alien system of mathematics is so badly erred in its fundamentals of arithmetic and consequently, primitive to such an extreme that it worries me. You alien mathematicians are not even aware of half of the phenomena in arithmetic and algebra that we Earth mathematicians are brilliant experts upon.**

**Mork: That is because our alien mathematicians learned, for the most part, to correctly distinguish between reality and fiction many tens of millennia ago. By the way, pure mathematics must not be synonymous with pure bullshit, just using math symbols to do it. We alien mathematicians as you call us in a derogatory way only work on what is to the best of our knowledge real yet we also have pure mathematics as well as applied mathematics. Ultimately, that is what makes the technology to build devices like my interstellar spaceship possible.**

**Now, correctly distinguishing between reality and fiction remains extremely difficult. It has been described as a neverending war for rationality that every individual alien mathematician and every generation must fight and make progress toward winning. Nevertheless, it is our prime directive that all of our alien mathematicians, regardless of their specialization, continuously work hardest on as it has been for nearly 50,000 years.**
**We put ourselves through this technical and conceptual Hell of endless, rigorous foundational and systemwide re-examinations because it is extremely important and its importance extends well past mathematics.**

**Mathematics is the most important thing in our civilization. Mathematics is the universal language running throughout all sciences and technologies. Technologies are what keep our "beings" from suffering in the wilderness like cavemen.**

**You mathematicians on Earth are either oblivious to the concept or too lazy to care.**

**Mindy: You came all of this way just to wrongly insult us? Listen,**
**I am an extremely good poker player. I can tell when people are bluffing. What you did not count on is that I can even tell when an ugly alien creature is bluffing. Nice speech but I know what is going on inside your mind?**

**By answering your questions, I inadvertently bragged about our Earthly achievements in mathematics and I made you feel inadequate and stupid. So, now you are lying like Hell about the stupendous levels of mathematics and technology prevalent to your alien civilization while thinking I have no way of seeing any evidence to the contrary. Frankly, I am amazed that "tin can" you are flying ever made it to Earth safely.**

**Mork: Can't you notice that your system of arithmetic is imbalanced and asymmetrical? After all, you have two real number, square roots of +1 yet zero real number, square roots of –1. In our alien system of arithmetic, we have one real number square root of +1 and one real number square root of –1 and they are +1 & –1, respectively.**

**Mindy: No. It seems perfectly logical to me. It is not broken. Therefore, it does not need to be fixed. Let me make it clear to you: On Earth, we don't really give a damned about balance, symmetry, consistency, conciseness, neatness or conceptual clarity in mathematics.**
**You have some sort of pointless neat freak or obsessive-compulsive disorder about mathematics that is annoying. Good luck in overcoming your many psychological problems, sincerely. No wonder you are incapable of mustering the intellect to understand or appreciate our system of mathematics.**

**Besides, that is what all of our textbooks worldwide say and all of our smartest mathematicians agree. Our mathematicians are not being arrogant when they claim to be the smartest people on our planet. They are just being truthful. In my studied opinion, I agree and I believe them. Apparently, they would be the smartest people on your planet, too. What do you think?**

**Mork: I think it is time for me to leave.**

**Mindy: Don't you want to talk with one of our expert mathematicians and learn a lot more before you leave?**

**Mork: No, thank you. I have heard enough.**

**Mindy: Well, you aren't going to learn much if you are in too big of a hurry.**

**Mork: I am not in a hurry at all. It took me 15 boring years to get here. It will take me 15 boring years to get back home.**

**Mindy: Your loss! Suit yourself. Although you are insufferably stupid, we don't have many visitors from outside our solar system. When will we see you again?**

**Mork: You won't … unless I return to destroy your planet.**

---

**Author's note: I envy Mork because he had the option to leave this planet for a more advanced civilization.**

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**Mork's report**

**I guess I am fortunate that the Earth people did not kill me and barbeque me considering how primitive, arrogant, prejudice and stupid-to-the-scary-extreme they are.**

**The conversation on mathematics did not and could not progress beyond basic arithmetic where they are hopelessly incompetent, irrational, closed-minded and self-justified. In fact, I believe my previous remark is inadequate or not emphatic enough to convey how astonishingly bad the current condition of mathematics on Earth is.**

**Apparently, their best mathematicians do <u>not</u> know the correct answer to:**

$$-1 \times -1 = n$$

**I hope I did not cause you to fall out of your chair!**

**Anyway, they are certain the correct answer is "+1" instead of "–1" (obviously).**

**I cannot easily express how frustrating and insulting my brief conversation with the science official on Earth was. Furthermore, I was surrounded by groups of people who were pointing machine guns at me throughout my entire time outside my spaceship. Still, I take full responsibility and apologize for spending inadequate time there, esp. since I wasted 30 years just traveling to and from their solar system.**

**Needless to say, their system of arithmetic and algebra is messed-up as badly as you probably cannot imagine if you are sane or educated yet in a bizarre way, they confidently and stubbornly believe that it has attained a condition that is miraculously inerrant and perfect … unearned.**

**[Yes, I am speaking of the same ideal, perfect condition our "alien" mathematicians have relentlessly worked hard to achieve for appr. 50,000 years which remains partially out-of-reach and probably will forever to some extent.]**

**They exercise no self-discipline in practicing the scientific method, make little successful effort to distinguish between mathematical reality and fiction and have no understanding or respect for the importance of symmetry and its guiding principles.**

**Apparently, their systems of knowledge throughout some natural sciences are nearly as corrupt, stagnant and unjust as their shockingly unenlightened political, legal, economic and social systems. This situation is not likely to change for the better in the foreseeable future.**

**Although it would be absolutely no loss to the galactic civilization and culture, I do not seriously recommend or see any point to destroying their pathetic, powerless planet. So, please do not take any military action based upon what I sarcastically said to the Earth woman in parting? Wait at least 500 years to see if they are making any significant progress. Meantime, my travel advisory is don't go there. I give it a one-star rating (on a scale of zero to five stars) only because they did not harm or kill me.**

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# fundamentals

## part II

**special definitions**

---

**opposition**

**The single unary operation in which any positive or negative real number (n) is transformed into the only real number of the opposite sign that, if added to it, would equal 0 (zero). In other words, its sign of quality is reversed but its absolute value remains exactly the same. The function-derived value is its opposite ($\wedge$n).**

**reciprocation**

**The single unary operation in which any positive or negative real number (n) Is transformed into the only real number of the same sign that, if multiplied by it, would equal +1 or –1, respectively. The function-derived value is its reciprocal ($\vee$n).**

**opposition <u>and</u> reciprocation**

**The dual unary operation in which any positive or negative real number (n) is transformed via opposition and reciprocation. The function-derived value is its opposite <u>and</u> reciprocal ($\wedge\vee$n).**

**identical multipliers**

**In revised multiplication, pairs of unique real number multipliers (b) that can multiply a single multiplicand (a) to equal products (c) identical to each other. A pair of second variables (b) that are opposites of one another have an equivalent effect upon the first variable (a). Thus, within this term "identical" is not intended literally but descriptively and only within the dynamic context of revised multiplication. An unknown identical multiplier can be determined where the other one is known by using the identical multiplier formula. Numerically, identical multipliers and identical exponents are equivalent.**

---

**identical exponents**

**In revised involution, pairs of unique real number exponents (b) that can involute a single base (a) to equal powers (c) identical to each other. A pair of second variables (b) that are opposites of one another have an equivalent effect upon the first variable (a). Thus, within this term "identical" is not intended literally but descriptively and only within the dynamic context of revised involution. An unknown identical exponent can be determined where the other one is known by using the identical exponent formula. Numerically, identical multipliers and identical exponents are equivalent.**

**missing variable**

**Within any of the three revised binary operations, whichever one of the three variables (“a”, “b” or “c”) that is unknown where the other two are known. The missing variable can always be determined by using the appropriate missing variable formula.**

**scope**

**Within any of the three revised binary operations, it refers to the fraction of the real number set to which the third variable “c” is bound considerate of the given domain of the first variable “a” and the given range of the second variable “b”.**

**correspondent notation**

**In revised algebra involving revised multiplication or revised involution, it is an algebraic notation which defines dual, mutually-exclusive relations between the signs of two unknowns as a multiplicand (a) and a multiplier (b) or as a base (a) and an exponent (b), respectively.**

---

## important distinctions in terminology

| conventional | revised |
|---|---|
| addition (conventional) | addition (conventional) |
| subtraction | --- |
| multiplication, conventional | multiplication, revised |
| division | --- |
| involution, conventional | involution, revised |
| evolution | --- |
| | |
| sum (conventional) | sum (conventional) |
| product, conventional | product, revised |
| power, conventional | power, revised |
| logarithm, conventional | logarithm, revised |
| | |
| --- | opposition |
| --- | reciprocation |
| --- | opposition and reciprocation |
| | |
| real number system (1-D), conventional | real number system (n-D), revised |
| | |
| the unit imaginary number | --- |
| complex number system (2-D) | --- |
| quaternion number system (4-D) | --- |
| octonion number system (8-D) | --- |
| sedenion number system (16-D) | --- |
| pathion number system (32-D) | --- |
| chingon number system (64-D) | --- |
| routon number system (128-D) | --- |
| voudon number system (256-D) | --- |
| | |
| hyper-complex voudon number system (512-D) | --- |
| | --- |
| | |
| hyper-quaternion voudon number system (1024-D) | --- |
| | --- |

| | |
|---|---|
| **arithmetic, conventional**<br>**algebra, conventional**<br>**geometry (conventional)**<br>**analytic geometry, conventional** | **arithmetic, revised**<br>**algebra, revised**<br>**geometry (conventional)**<br>**analytic geometry, revised** |
| **trigonometry (conventional)**<br>**trigonometric ratios (conventional)** | **trigonometry (conventional)**<br>**trigonometric ratios (conventional)** |
| **tangent ratio (conventional)**<br>**sine ratio (conventional)**<br>**cosine ratio (conventional)**<br>**cotangent ratio (conventional)**<br>**cosecant ratio (conventional)**<br>**secant ratio (conventional)** | **tangent ratio (conventional)**<br>**sine ratio (conventional)**<br>**cosine ratio (conventional)**<br>**cotangent ratio (conventional)**<br>**cosecant ratio (conventional)**<br>**secant ratio (conventional)** |
| **analytic trigonometry, conventional** | **analytic trigonometry, revised** |
| **trigonometric functions, conventional** | **trigonometric functions, revised** |
| **tangent function, conventional**<br>**sine function (conventional)**<br>**cosine function (conventional)**<br>**cotangent function, conventional**<br>**cosecant function (conventional)**<br>**secant function (conventional)** | **tangent function, revised**<br>**sine function (conventional)**<br>**cosine function (conventional)**<br>**cotangent function, revised**<br>**cosecant function (conventional)**<br>**secant function (conventional)** |
| **circular functions, conventional** | **circular functions, revised** |
| **calculus, conventional** | **calculus, revised** |
| **linear equations, conventional**<br>**linear functions, conventional** | **linear equations, revised**<br>**linear functions, revised** |
| **slope, conventional**<br>**angle of inclination (conventional)** | **slope, revised**<br>**angle of inclination (conventional)** |
| **power functions, conventional** | **power functions, revised** |
| **exponential functions, conventional**<br>**logarithmic functions, conventional** | **exponential functions, revised**<br>**logarithmic functions, revised** |
| **derivative, conventional**<br>**anti-derivative, conventional** | **derivative, revised**<br>**anti-derivative, revised** |

**symbols**

| | |
|---|---|
| **addition** | **+** |
| **multiplication, revised** | **x** |
| **involution, revised** | $a^b$ |
| **opposition** | $\wedge$ |
| **reciprocation** | $\vee$ |
| **opposition <u>and</u> reciprocation** | $\wedge\vee$ |
| **positive** | **+** |
| **negative** | **–** |
| **an unknown variable** | **x** |
| **a real number/scalar** | **n** |
| **the set of real numbers** | **R** |
| **an element of set ...** | $\in$ |
| **not an element of set ...** | $\notin$ |
| **intersection** | $\cap$ |
| **union** | $\cup$ |
| **equal** | **=** |
| **not equal** | $\neq$ |
| **appr. equal** | $\cong$ |
| **greater than** | **>** |
| **lesser than** | **<** |

| | |
|---|---|
| **the exponential constant** | |
| **- positive** | **+e** |
| **- negative** | **–e** |
| **the unit imaginary number** | **i** |
| **positive over negative** | ± |
| **negative over positive** | ∓ |
| **positive infinity** | **+∞** |
| **negative infinity** | **–∞** |
| **positive infinitesimal** | ∨**+∞** |
| **negative infinitesimal** | ∨**–∞** |
| **revised slope** | **m** |
| **revised angle of inclination** | θ |
| **revised tangent function** | **tan*** |
| **revised cotangent function** | **cot*** |
| **absolute value** | **\|n \|** |
| **equivalent equations** | ≡ |
| **greater than or equal to** | ≥ |
| **lesser than or equal to** | ≤ |

**Notes-**

1. **In the conventional numerical system, a negative symbol (–) has three distinctly different usages and meanings:**

   **- to indicate the sign of a real number as negative.**

   **- to indicate the conventional binary operation of subtraction.**

   **- to indicate the unary operation of opposition in conventional notation.**

---

2. **In the revised numerical system, a negative symbol (–) has one exclusive usage and meaning:**

   **- to indicate the sign of a real number as negative.**

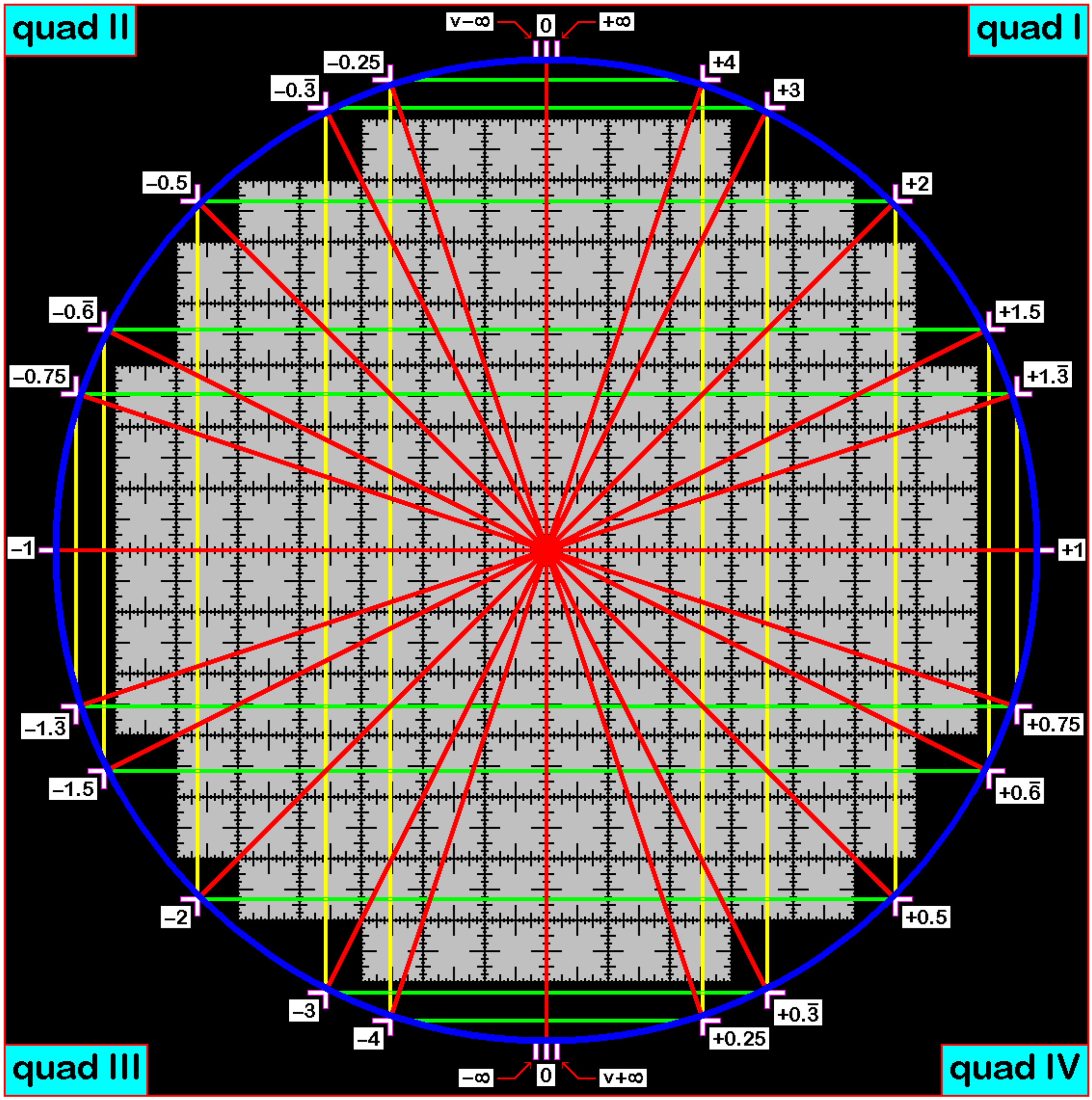
---

**There is no revised binary operation of subtraction.**

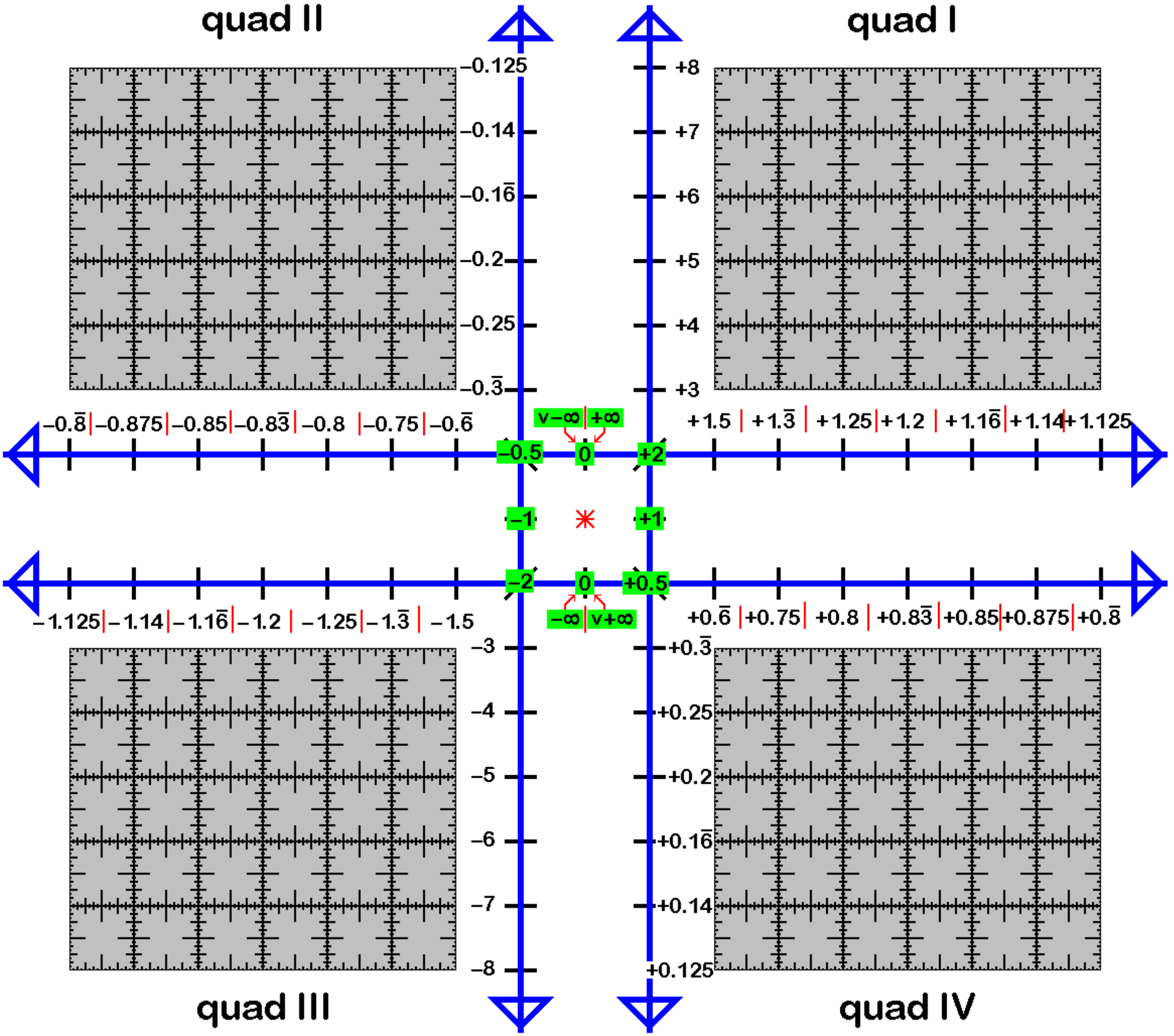
**The unary operation of opposition is indicated by the opposition symbol (∧) instead in revised notation.**

---

# the extended real number continuum



# the extended real number continuum



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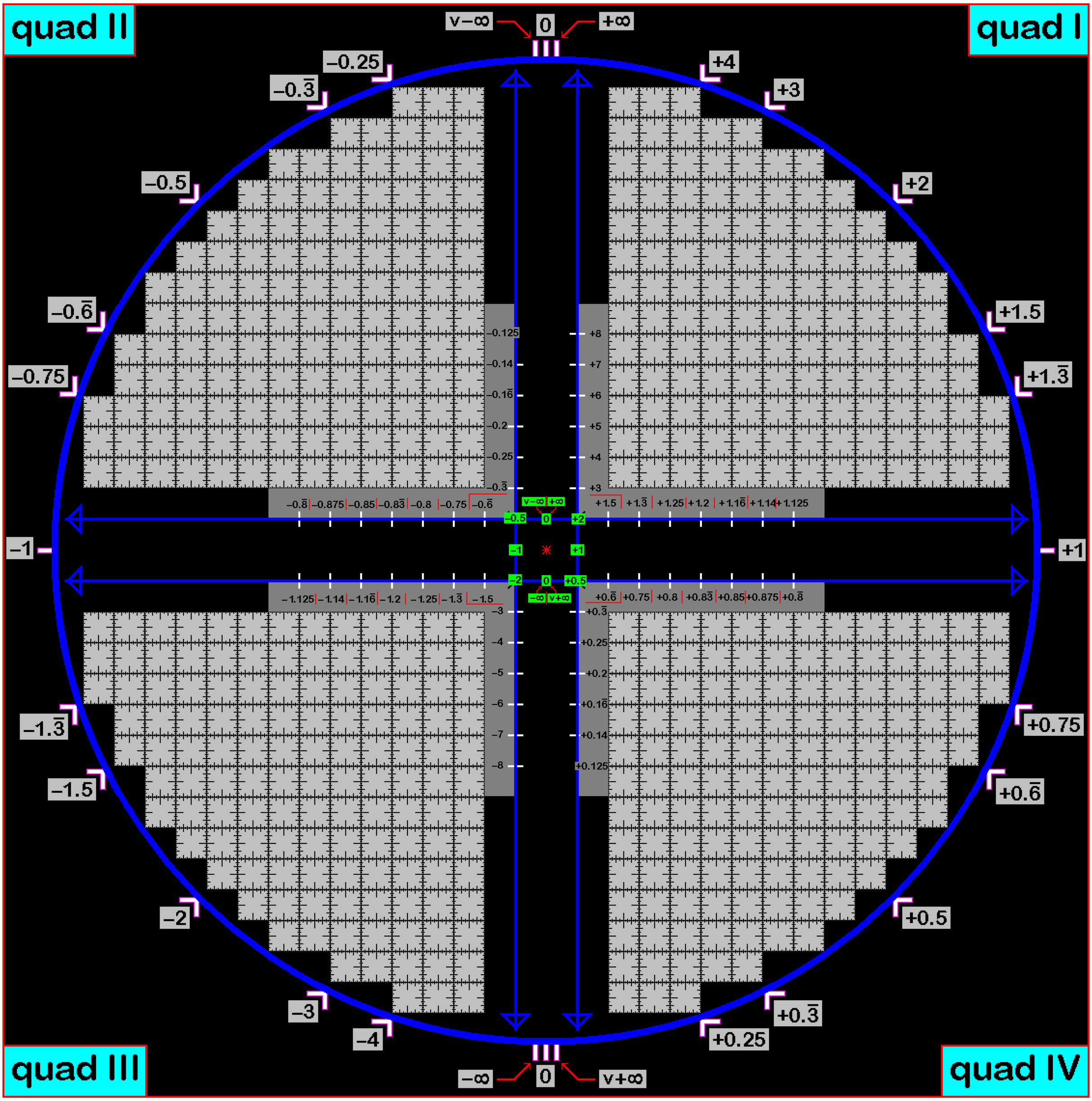
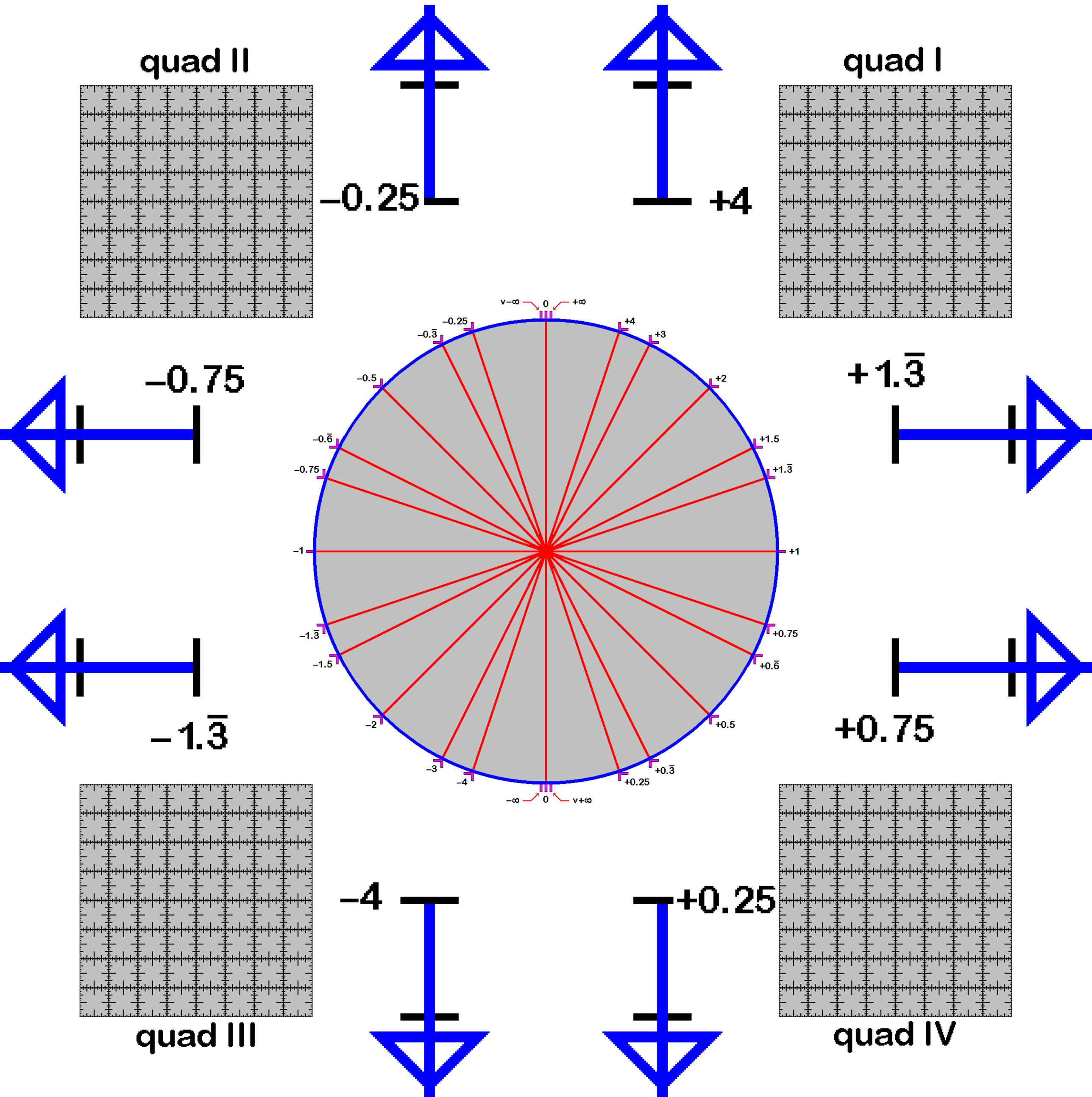


# the extended real number continuum



**the extended real number continuum**
**(including the revised slope system)**

**circular and/or linear depictions (4)**

**- legend**

---

**The four graphs are all optional, equally-valid representations of the single, unified reality of the extended real number continuum.**

**The circular depiction and the linear depiction are simple, one-part models. The circular-linear depiction and the linear-circular depiction are compound, two-part models.**

**The circular-linear depiction has the circular depiction encompassing the linear depiction within it. The linear-circular depiction has the linear depiction encompassing the circular depiction within it.**

**Both two-part models are merely the first application of the fact that alternating circular and linear representations where circular depictions are placed inside or outside linear depictions (and vice versa) are infinitely progressive <u>and</u> infinitely regressive in scale and thus, infinite-part models are constructible in theory that also represent the extended real number continuum.**

---

**geometrical interrelations-**

| | |
|---|---|
| **opposition** | **diameter** |
| **reciprocation** | **vertical line** |
| **opposition <u>and</u> reciprocation** | **horizontal line** |

---

**Any two points that are intersected by a line that is horizontal, vertical or a diameter have the indicated unary operation between each other numerically.**

---

**quadrant I** $+n > +1$

**all positive, extended real numbers greater than +1**

**quadrant II** $-n > -1$

**all negative, extended real numbers greater than –1**

**quadrant III** $-n < -1$

**all negative, extended real numbers lesser than –1**

**quadrant IV** $+n < +1$

**all positive, extended real numbers lesser than +1**

---

| | |
|---|---|
| **quadrants I, IV** | **all positive, extended real numbers** |
| **quadrants II, III** | **all negative, extended real numbers** |
| **quadrants I, III** | **all absolute values greater than +1** |
| **quadrants II, IV** | **all absolute values lesser than +1** |

---

**special symbols-**

$+\infty$ **= positive infinity**

$\vee+\infty$ **= positive infinitesimal;**
**the reciprocal of positive infinity**

$-\infty$ **= negative infinity**

$\vee-\infty$ **= negative infinitesimal;**
**the reciprocal of negative infinity**

---

**quarter-points**

| | |
|---|---|
| **quadrant I-II ray** | **0 [zero (high)]** |
| **quadrant II-III ray** | **–1** |
| **quadrant III-IV ray** | **0 [zero (low)]** |
| **quadrant IV-I ray** | **+1** |

---

**half-points**

| | |
|---|---|
| **quadrant I-II & III-IV line**<br>**(quadrant I-II ray & quadrant III-IV ray)** | **0 (zero)** |
| **quadrant II-III & IV-I line**<br>**(quadrant II-III ray & quadrant IV-I ray)** | **±1 <u>or</u> ∓1** |

---

**centerpoint**

**The single point of intersection with the endpoints of all rays. Moreover, it is the exact centerpoint of all four graphs involving circular and/or linear depictions. It has no numerical value ascribed.**

---

**the revised slope system**

---

**With every ray (or 2-D position vector) sharing the centerpoint (having no numerical value) as its endpoint, the point(s) of intersection with the extended real number continuum gives the other of two points needed to define a ray. Points of intersection can lie on a circle, horizontal axis or vertical axis. [All are marked in dark blue in all four graphs.]**

**The revised slope of every ray (or 2-D position vector) in one quadrant is determined by and equals the positive <u>or</u> negative real number it intersects on the extended real number continuum.**

**Every unique ray has one unique revised slope. So, it is not possible for two unique rays to share the same revised slope. This important reform, present only in the revised slope system, prevents the confusion possible in the conventional slope system where two unique (opposite direction) rays share the same conventional slope.**

**In other words, conventional slopes recycle every 180° (a semi-circle) of a circle yielding two-fold duplication (which causes ambiguity and confusion) whereas revised slopes recycle every 360° (a full circle) of a circle yielding no duplication and correlating perfectly to a circle.**

**[Note: Revised arithmetic unavoidably alters all analytic/numerical branches of math. Since the slope system had to be changed anyway to accommodate revised analytic geometry and revised calculus, I decided I might as well improve it in a needed, beneficial way.]**

**The revised slopes of every line in two quadrants are determined by and equal the opposite positive <u>and</u> negative real numbers its two component rays intersect on the extended real number continuum.**

**This means every unique line consists of a unique pair of geometrically-opposite direction rays with revised slopes that correspondingly are numerically-opposite. So, it is not possible for two unique lines to share the same pair of revised slopes.**

---

**To be sure, the revised slope system wholly exists within the extended real number continuum. All rays (or 2-D position vectors) and lines should be drafted on it instead of the rectangular coordinate system (by traditional practice) where the formulae to calculate their revised slopes are much more complicated [not included within this work]. Furthermore, it is generally a superior, versatile framework and foundation for revised analytic geometry and revised analytic trigonometry that can accommodate and correlate geometric figures, trigonometric functions, unary operations, numbers, slopes, etc into formulae and equations in a great variety and multiplicity of ways (some, unprecedented) simultaneously. Despite its obvious, great potential, it has not yet been developed and explored in detail.**

**Note that although the extended real number continuum is obviously a fundamental, numerical-geometrical model that underlies and precedes arithmetic, it is also true that the revised slope system it contains, defines and is interchangeable with (since no different formulae are required) entails revised trigonometric functions and maps or creates the foundation of revised calculus.**

_____

---

**the extended real number continuum**

**extended real number - revised slope correlations**

---

**n = an extended real number on the extended real number continuum.**
**This value can lie on a circle, horizontal axis or vertical axis.**

**m = a revised slope within the extended real number continuum.**
**This value can lie on a circle, horizontal axis or vertical axis.**

**tan* = the revised tangent function.**

**$\theta$ = an angle of inclination.**

---

**n = m**

---

**quadrant I**

$$+n = \tan^* +\theta + +1$$
$$+m = \tan^* +\theta + +1$$

**quadrant II**

$$-n = \vee(\tan^* -\theta + -1)$$
$$-m = \vee(\tan^* -\theta + -1)$$

**quadrant III**

$$-n = \tan^* -\theta + -1$$
$$-m = \tan^* -\theta + -1$$

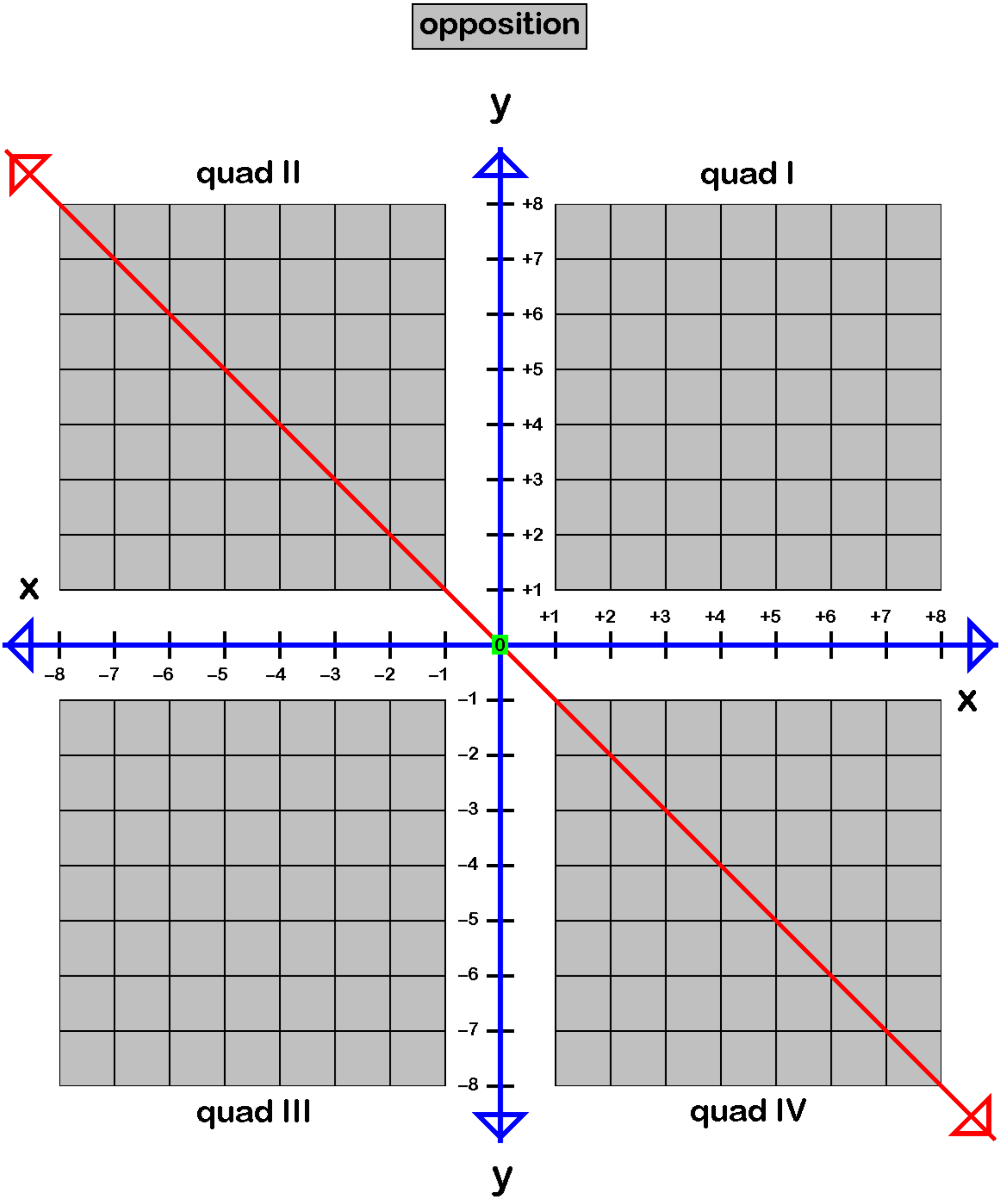
**quadrant IV**

$$+n = \vee(\tan^* +\theta + +1)$$
$$+m = \vee(\tan^* +\theta + +1)$$

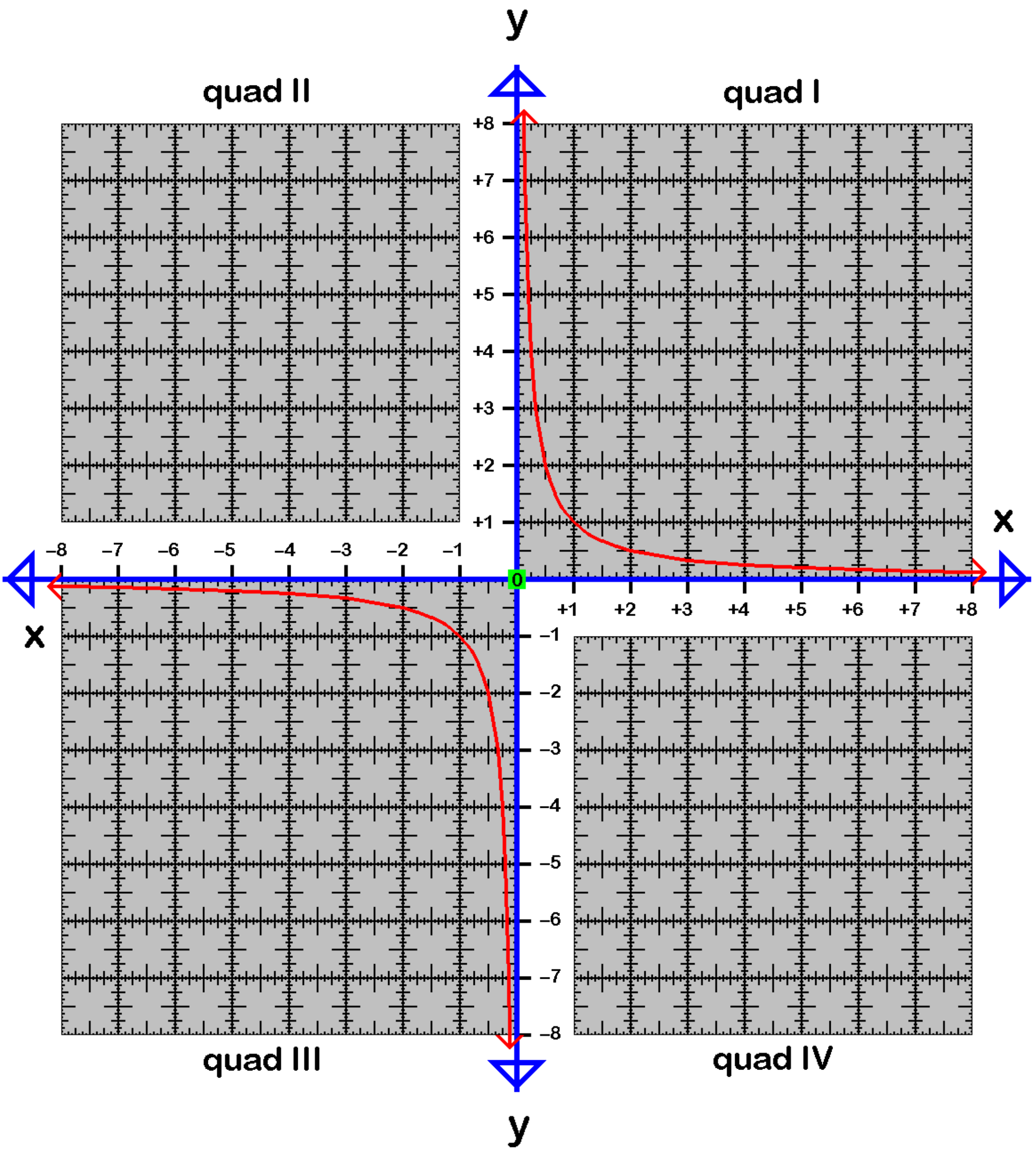
---

| **revised tangent function of angles of inclination** | |
|---|---|
| **positive values** | |
| **tan* +θ** | **+θ** |
| **0 (zero)** | **0°** |
| **$+0.1\overline{6}$** | **+9.5°** |
| **+0.2** | **+11.3°** |
| **+0.25** | **+14.0°** |
| **$+0.\overline{3}$** | **+18.5°** |
| **+0.5** | **+26.5°** |
| **+1** | **+45.0°** |
| **+2** | **+63.5°** |
| **+3** | **+71.5°** |
| **+4** | **+76.0°** |
| **+5** | **+78.7°** |
| **+6** | **+80.5°** |

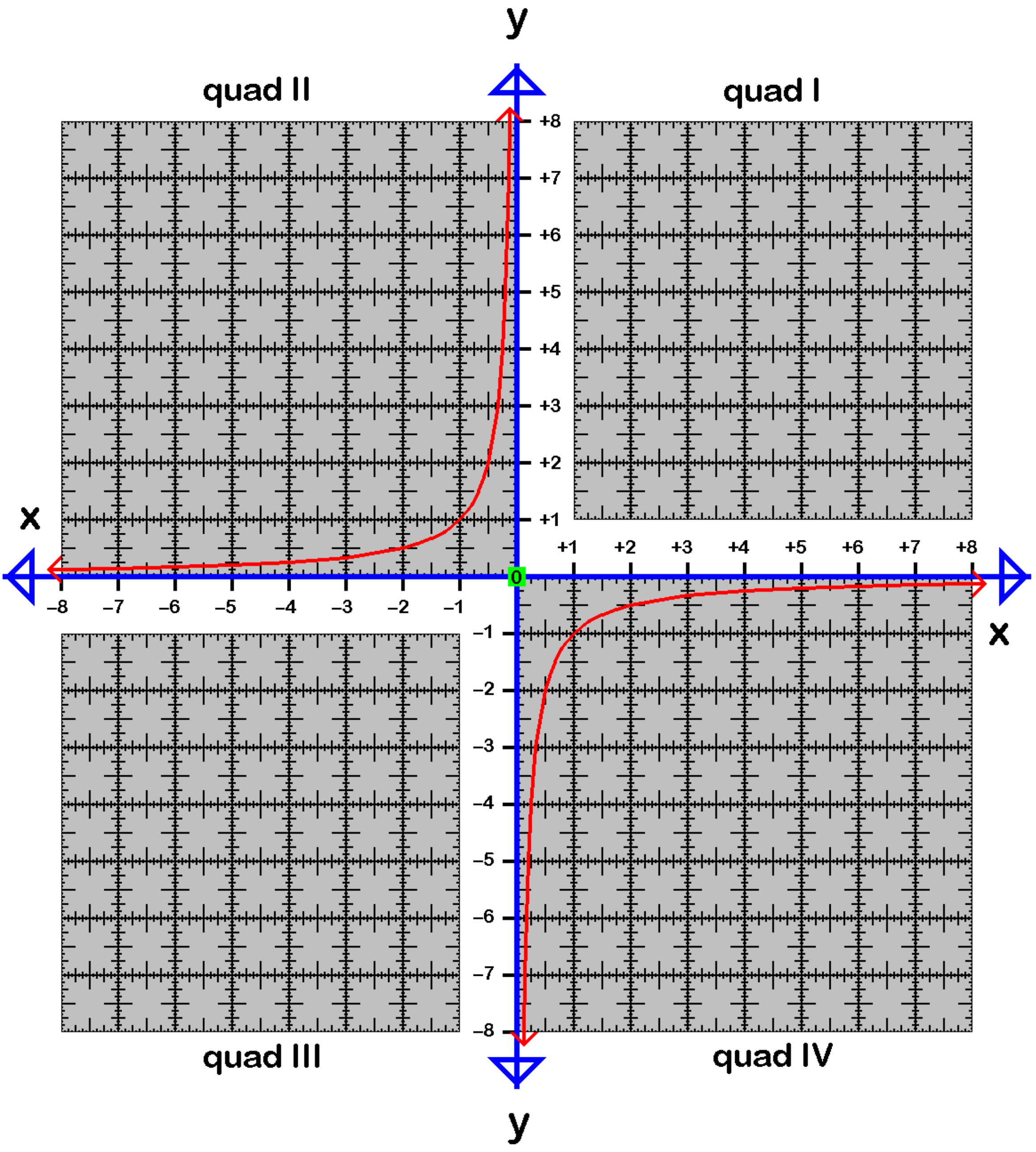
| | |
|---|---|
| **revised tangent function of angles of inclination** | |
| **negative values** | |
| **tan* –θ** | **–θ** |
| **0 (zero)** | **0°** |
| **$-0.1\overline{6}$** | **–9.5°** |
| **–0.2** | **–11.3°** |
| **–0.25** | **–14.0°** |
| **$-0.\overline{3}$** | **–18.5°** |
| **–0.5** | **–26.5°** |
| **–1** | **–45.0°** |
| **–2** | **–63.5°** |
| **–3** | **–71.5°** |
| **–4** | **–76.0°** |
| **–5** | **–78.7°** |
| **–6** | **–80.5°** |


opposition
y
quad II
quad I
+8
+7
+6
+5
+4
+3
+2
+1
x
+1 +2 +3 +4 +5 +6 +7 +8
0
−8 −7 −6 −5 −4 −3 −2 −1
x
−1
−2
−3
−4
−5
−6
−7
−8
quad III
quad IV
y

# reciprocation



# opposition and reciprocation



**unary operations**

**opposition and/or reciprocation**

**- legend**

---

**opposition**

$a = \wedge b$

$b = \wedge a$

---

$a + b = 0$

---

**a continuous function**

---

**reciprocation**

**positive reciprocation**

$+a = \vee +b$

$+b = \vee +a$

---

$+a \times +b = +1$

---

**negative reciprocation**

$-a = \vee -b$

$-b = \vee -a$

---

$-a \times -b = -1$

---

**a discontinuous function**

---

**opposition <u>and</u> reciprocation**

**positive-negative**

**+a = ∧∨–b**

**–b = ∧∨+a**

---

**negative-positive**

**–a = ∧∨+b**

**+b = ∧∨–a**

---

**a discontinuous function**

---

| | | |
|---|---|---|
| **conversions-**<br><br>**conventional equivalencies of opposition and/or reciprocation** | | |
| **universal formulae** | | |
| **functions** | **revised notation** | **conventional notation** |
| **opposition** | **∧n** | **–(n)** |
| **reciprocation** | **∨n** | **+1 ÷ n** |
| **opposition <u>and</u> reciprocation** | **∧∨n** | **–(+1 ÷ n)** |

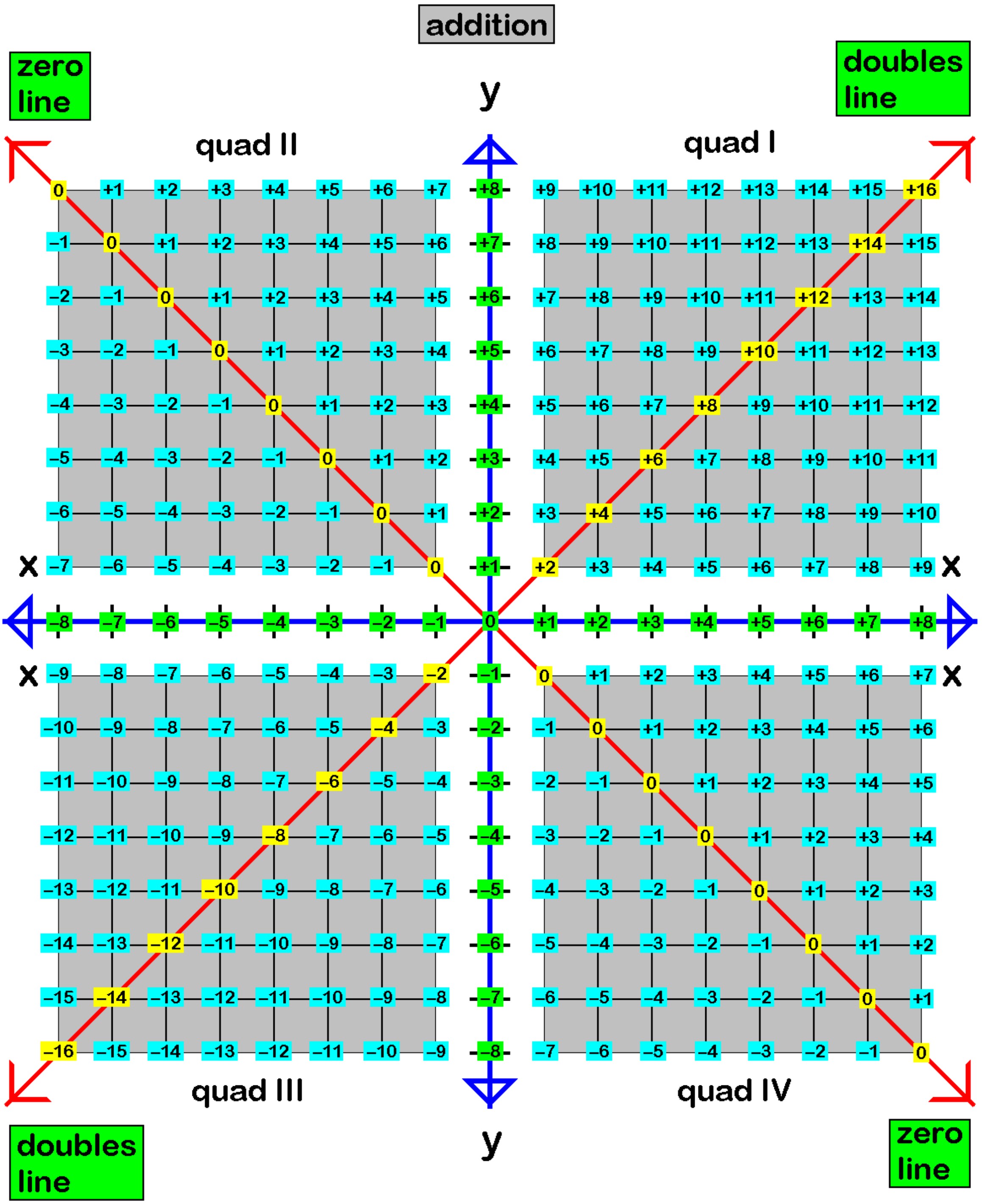
| | |
|---|---|
| **special cases which contrast between the two systems** | |
| **functions** | **revised notation** |
| **reciprocation** | $\vee$ **0 = 0** |
| **opposition <u>and</u> reciprocation** | $\wedge\vee$ **0 = 0** |

## unary operations demonstrations

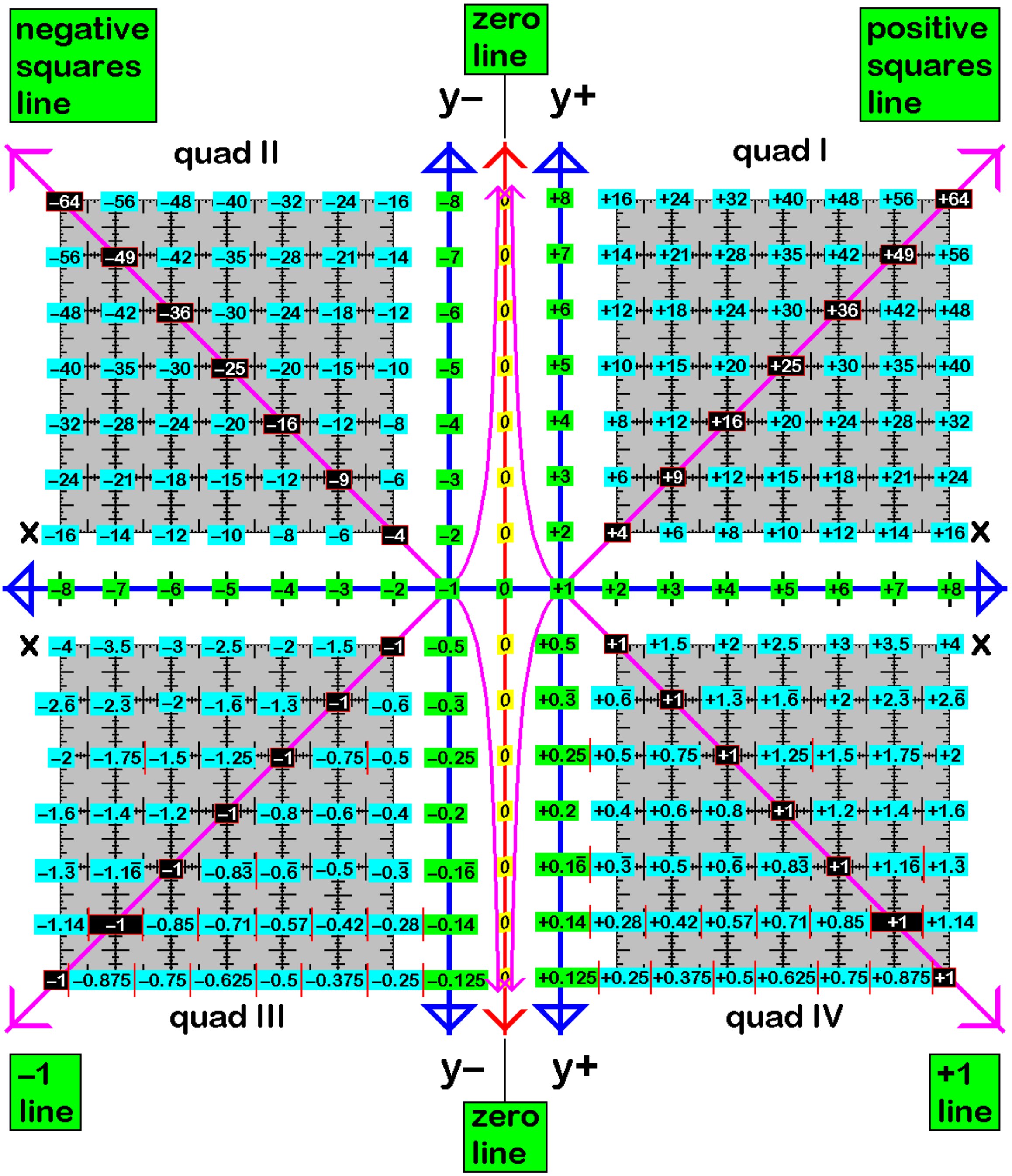
| opposition<br>f(n) = ∧(n) | reciprocation<br>f(n) = ∨(n) | opposition <u>and</u> reciprocation<br>f(n) = ∧∨(n) |
|---|---|---|
| ∧+4 = –4 | ∨+4 = +0.25 | ∧∨+4 = –0.25 |
| ∧+0.5 = –0.5 | ∨+0.5 = +2 | ∧∨+0.5 = –2 |
| ∧–0.2 = +0.2 | ∨–0.2 = –5 | ∧∨–0.2 = +5 |
| ∧–2 = +2 | ∨–2 = –0.5 | ∧∨–2 = +0.5 |
| ∧0 = 0 | ∨0 = 0 | ∧∨0 = 0 |
| ∧+1 = –1 | ∨+1 = +1 | ∧∨+1 = –1 |
| ∧–1 = +1 | ∨–1 = –1 | ∧∨–1 = +1 |

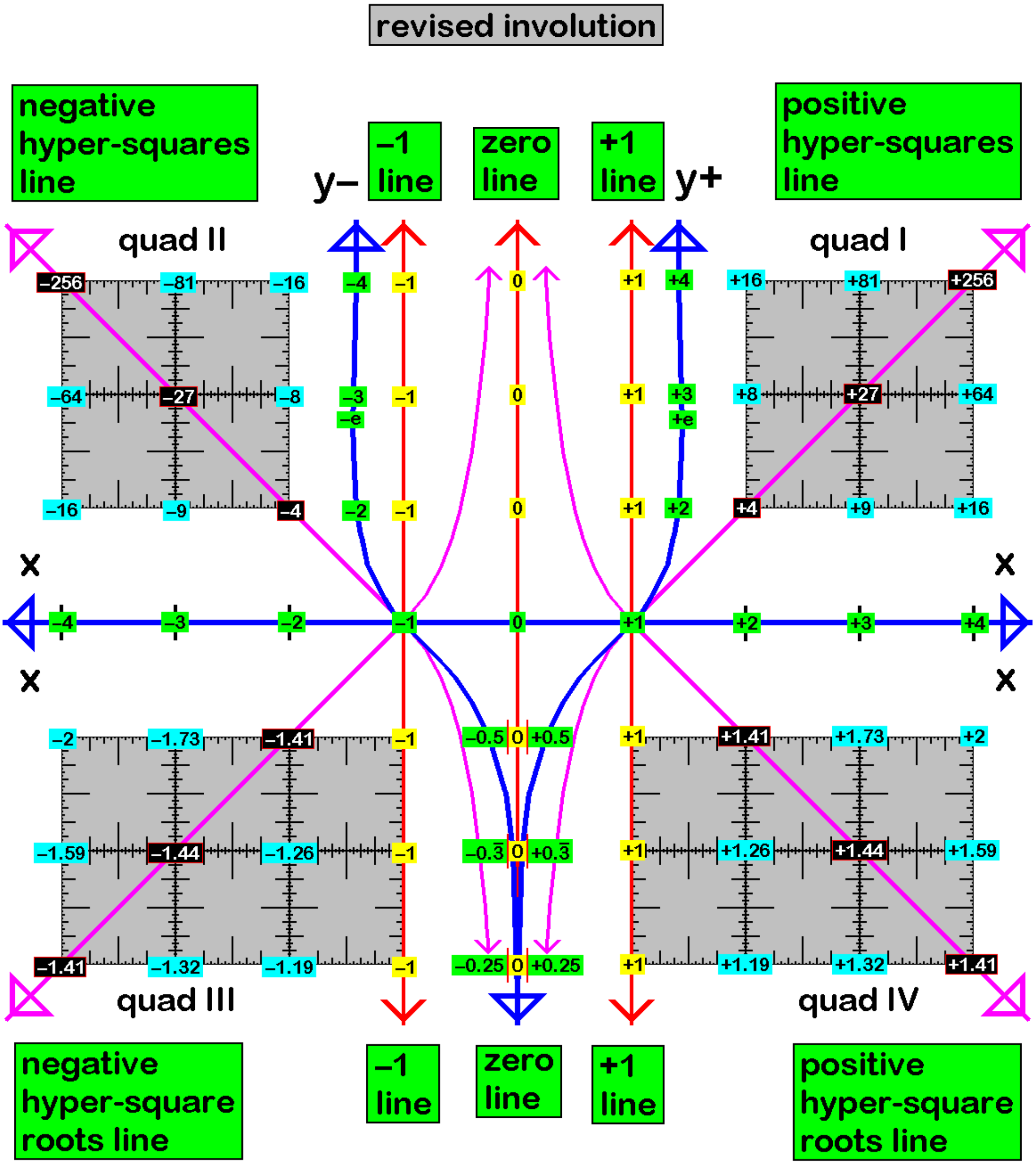
# revised arithmetic and algebra

## part III


addition
zero line
doubles line
y
quad II
quad I
x
x
quad III
quad IV
doubles line
y
zero line

# revised multiplication




revised involution
negative hyper-squares line
y–
–1 line
zero line
+1 line
y+
positive hyper-squares line
quad II
quad I
–256
–81
–16
–64
–27
–8
–16
–9
–4
–4
–3
–e
–2
–1
–1
–1
0
0
0
+1
+1
+1
+1
+4
+3
+e
+2
+16
+81
+256
+8
+27
+64
+16
+9
+4
–4
–3
–2
–1
0
+1
+2
+3
+4
–2
–1.73
–1.41
–1
–0.5
0
+0.5
+1
+1.41
+1.73
+2
–1.59
–1.44
–1.26
–1
–0.3̄
0
+0.3̄
+1
+1.26
+1.44
+1.59
–1.41
–1.32
–1.19
–1
–0.25
0
+0.25
+1
+1.19
+1.32
+1.41
quad III
quad IV
negative hyper-square roots line
–1 line
zero line
+1 line
positive hyper-square roots line

**midpoints and origins**

---

**addition**

**midpoints**

| | |
|---|---|
| **x axis** | **0 (zero)** |
| **y axis** | **0 (zero)** |

**origin**

| | |
|---|---|
| **x y axes** | **(0, 0)** |

**revised multiplication**

**midpoints**

| | |
|---|---|
| **x axis** | **0 (zero)** |
| **y+ axis** | **+1** |
| **y- axis** | **-1** |

**origins**

| | |
|---|---|
| **x y+ axes** | **(+1, +1)** |
| **x y- axes** | **(-1, -1)** |

**revised involution**

**midpoints**

| | |
|---|---|
| **x axis** | **0 (zero)** |
| **y+ axis** | **+1** |
| **y- axis** | **-1** |

**origins**

| | |
|---|---|
| **x y+ axes** | **(+1, +1)** |
| **x y- axes** | **(-1, -1)** |

## quadrant definitions

| | addition | revised multiplication | revised involution |
|---|---|---|---|
| **quad** | | | |
| **I** | **(x > 0) ∩ y > 0)** | **(x > 0) ∩ (y+ > +1)**<br>**(x > 0) ∩ (y– < –1)** | **(x > 0) ∩ (y+ > +1)**<br>**(x > 0) ∩ (y– < –1)** |
| **II** | **(x < 0) ∩ (y > 0)** | **(x < 0) ∩ (y+ > +1)**<br>**(x < 0) ∩ (y– < –1)** | **(x < 0) ∩ (y+ > +1)**<br>**(x < 0) ∩ (y– < –1)** |
| **III** | **(x < 0) ∩ (y < 0)** | **(x < 0) ∩ (y+ < +1)**<br>**(x < 0) ∩ (y– > –1)** | **(x < 0) ∩ (y+ < +1)**<br>**(x < 0) ∩ (y– > –1)** |
| **IV** | **(x > 0) ∩ (y < 0)** | **(x > 0) ∩ (y+ < +1)**<br>**(x > 0) ∩ (y– > –1)** | **(x > 0) ∩ (y+ < +1)**<br>**(x > 0) ∩ (y– > –1)** |

**variables**
**revised binary operations**

---

**addition**

| | | |
|---|---|---|
| **a** | **augend**<br>**summand, first** | **on x axis**<br>**(x coordinate- abscissa)** |
| **b** | **addend**<br>**summand, second** | **on y axis**<br>**(y coordinate- ordinate)** |
| **c** | **sum** | **on x y axes plane** |

**revised multiplication**

| | | |
|---|---|---|
| **a** | **multiplicand**<br>**factor, first** | **on x axis**<br>**(x coordinate- abscissa)** |
| **b** | **multiplier**<br>**factor, second** | **on y± axis**<br>**(y± coordinate- ordinate)** |
| **c** | **revised product** | **on x y± axes plane** |

**revised involution**

| | | |
|---|---|---|
| **a** | **base** | **on x axis**<br>**(x coordinate- abscissa)** |
| **b** | **exponent** | **on y± axes**<br>**(y± coordinate- ordinate)** |
| **c** | **revised power** | **on x y± axes plane** |

---

| conversions-<br><br>conventional equivalencies of revised binary operations | revised notation | conventional notation | comparisons |
|---|---|---|---|
| addition | +a + +b | +a + +b | identical |
| | +a + –b | +a + –b | identical |
| | –a + +b | –a + +b | identical |
| | –a + –b | –a + –b | identical |
| multiplication | +a x +b | +a x +b | identical |
| | +a x –b | +a x +b | different |
| | –a x +b | – (+a x +b) | identical |
| | –a x –b | – (+a x +b) | different |
| involution | $+a^{+b}$ | $+a^{+b}$ | identical |
| | $+a^{-b}$ | $+a^{+b}$ | different |
| | $-a^{+b}$ | $-(+a^{+b})$ | different |
| | $-a^{-b}$ | $-(+a^{+b})$ | different |

| special cases which contrast between the two systems | |
|---|---|
| **binary operations** | **revised notation** |
| **involution** | $+a^{0} = 0$<br>$-a^{0} = 0$<br>$0^{0} = 0$ |
| | |

**summary**

**Addition is identical in every respect under both conventional arithmetic and revised arithmetic. Thus, no distinction is made between revised and conventional counterparts. Addition within revised arithmetic is implicitly a conventional binary operation.**

**Multiplication is identical under both conventional and revised arithmetic in two situations:**

1. **Where both the multiplicand (a) and the multiplier (b) are positive real numbers.**

2. **Where the multiplicand (a) is a negative real number and the multiplier (b) is a positive real number.**

**[total of ½ of all possible interactions]**

**Otherwise, revised multiplication and its revised products (c) are uniquely different from their conventional counterparts [½ of all possible interactions] although they have the same absolute values.**

**Involution is identical under both conventional and revised arithmetic <u>only</u> where both the base (a) and the exponent (b) are positive real numbers [¼ of all possible interactions]. Otherwise, revised involution and its revised powers (c) are uniquely different from their conventional counterparts [¾ of all possible interactions].**

## revised binary operations demonstrations

| addition | revised multiplication | revised involution |
|---|---|---|
| +2 + +4 = +6 | +2 × +4 = +8 | $+2^{+4} = +16$ |
| +2 + +1 = +3 | +2 × +1 = +2 | $+2^{+1} = +2$ |
| +2.0 + +0.5 = +2.5 | +2.0 × +0.5 = +1 | $+2^{+0.5} = +1.4142 \ldots$ |
| +2 + 0 = +2 | +2 × 0 = 0 | $+2^{0} = 0$ |
| +2.0 + –0.5 = +1.5 | +2.0 × –0.5 = +1 | $+2^{-0.5} = +1.4141 \ldots$ |
| +2 + –1 = +1 | +2 × –1 = +2 | $+2^{-1} = +2$ |
| +2 + –4 = –2 | +2 × –4 = +8 | $+2^{-4} = +16$ |
| | | |

| | | |
|---|---|---|
| $+1 + +4 = +5$<br>$+1 + +1 = +2$<br>$+1.0 + +0.5 = +1.5$<br>$+1 + 0 = +1$<br>$+1.0 + -0.5 = +0.5$<br>$+1 + -1 = 0$<br>$+1 + -4 = -3$ | $+1 \times +4 = +4$<br>$+1 \times +1 = +1$<br>$+1.0 \times +0.5 = +0.5$<br>$+1 \times 0 = 0$<br>$+1.0 \times -0.5 = +0.5$<br>$+1 \times -1 = +1$<br>$+1 \times -4 = +4$ | $+1^{+4} = +1$<br>$+1^{+1} = +1$<br>$+1^{+0.5} = +1$<br>$+1^{0} = 0$<br>$+1^{-0.5} = +1$<br>$+1^{-1} = +1$<br>$+1^{-4} = +1$ |
| $+0.25 + +4.00 = +4.25$<br>$+0.25 + +1.00 = +1.25$<br>$+0.25 + +0.50 = +0.75$<br>$+0.25 + 0 = +0.25$<br>$+0.25 + -0.50 = -0.25$<br>$+0.25 + -1.00 = -0.75$<br>$+0.25 + -4 = -3.75$ | $+0.25 \times +4.00 = +1$<br>$+0.25 \times +1.00 = +0.25$<br>$+0.25 \times +0.50 = +0.125$<br>$+0.25 \times 0 = 0$<br>$+0.25 \times -0.50 = +0.125$<br>$+0.25 \times -1.00 = +0.25$<br>$+0.25 \times -4.00 = +1$ | $+0.25^{+4} = +0.0039 \ldots$<br>$+0.25^{+1} = +0.25$<br>$+0.25^{+0.5} = +0.5$<br>$+0.25^{0} = 0$<br>$+0.25^{-0.5} = +0.5$<br>$+0.25^{-1} = +0.25$<br>$+0.25^{-4} = +0.0039 \ldots$ |
| | | |

| | | |
|---|---|---|
| $0 + +4 = +4$<br>$0 + +1 = +1$<br>$0 + +0.5 = +0.5$<br>$0 + 0 = 0$<br>$0 + -0.5 = -0.5$<br>$0 + -1 = -1$<br>$0 + -4 = -4$ | $0 \times +4 = 0$<br>$0 \times +1 = 0$<br>$0 \times +0.5 = 0$<br>$0 \times 0 = 0$<br>$0 \times -0.5 = 0$<br>$0 \times -1 = 0$<br>$0 \times -4 = 0$ | $0^{+4} = 0$<br>$0^{+1} = 0$<br>$0^{+0.5} = 0$<br>$0^{0} = 0$<br>$0^{-0.5} = 0$<br>$0^{-1} = 0$<br>$0^{-4} = 0$ |
| $-0.25 + +4.00 = +3.75$<br>$-0.25 + +1.00 = +0.75$<br>$-0.25 + +0.50 = +0.25$<br>$-0.25 + 0 = -0.25$<br>$-0.25 + -0.50 = -0.75$<br>$-0.25 + -1.00 = -1.25$<br>$-0.25 + -4 = -4.25$ | $-0.25 \times +4.00 = -1$<br>$-0.25 \times +1.00 = -0.25$<br>$-0.25 \times +0.50 = -0.125$<br>$-0.25 \times 0 = 0$<br>$-0.25 \times -0.50 = -0.125$<br>$-0.25 \times -1.00 = -0.25$<br>$-0.25 \times -4.00 = -1$ | $-0.25^{+4} = -0.0039 \ldots$<br>$-0.25^{+1} = -0.25$<br>$-0.25^{+0.5} = -0.5$<br>$-0.25^{0} = 0$<br>$-0.25^{-0.5} = -0.5$<br>$-0.25^{-1} = -0.25$<br>$-0.25^{-4} = -0.0039 \ldots$ |
| | | |

| | | |
|---|---|---|
| $-1 + +4 = +3$<br>$-1 + +1 = 0$<br>$-1.0 + +0.5 = -0.5$<br>$-1 + 0 = -1$<br>$-1.0 + -0.5 = -1.5$<br>$-1 + -1 = -2$<br>$-1 + -4 = -5$ | $-1 \times +4 = -4$<br>$-1 \times +1 = -1$<br>$-1.0 \times +0.5 = -0.5$<br>$-1 \times 0 = 0$<br>$-1.0 \times -0.5 = -0.5$<br>$-1 \times -1 = -1$<br>$-1 \times -4 = -4$ | $-1^{+4} = -1$<br>$-1^{+1} = -1$<br>$-1^{+0.5} = -1$<br>$-1^{0} = 0$<br>$-1^{-0.5} = -1$<br>$-1^{-1} = -1$<br>$-1^{-4} = -1$ |
| $-2 + +4 = +2$<br>$-2 + +1 = -1$<br>$-2.0 + +0.5 = -1.5$<br>$-2 + 0 = -2$<br>$-2.0 + -0.5 = -2.5$<br>$-2 + -1 = -3$<br>$-2 + -4 = -6$ | $-2 \times +4 = -8$<br>$-2 \times +1 = -2$<br>$-2.0 \times +0.5 = -1$<br>$-2 \times 0 = 0$<br>$-2.0 \times -0.5 = -1$<br>$-2 \times -1 = -2$<br>$-2 \times -4 = -8$ | $-2^{+4} = -16$<br>$-2^{+1} = -2$<br>$-2^{+0.5} = -1.4142 \ldots$<br>$-2^{0} = 0$<br>$-2^{-0.5} = -1.4142 \ldots$<br>$-2^{-1} = -2$<br>$-2^{-4} = -16$ |

**the meaning and use of correspondent notation**

---

**Although correspondent notation is foreign to conventional algebra, it is basic to revised algebra. The sharp contrasts between revised multiplication and conventional multiplication as well as revised involution and conventional involution underlie the substantial differences between revised algebra and conventional algebra.**

**In conventional algebra, the following relation holds between conventional multiplication and addition (conventional)-**

**arbitrary example**

$$x + x + x = (+3)(x)$$

**Clearly, each self-addition is implicitly counted as "+1" additions until the number of the coefficient (e.g., "+3") is reached. This is the positive number bias of conventional arithmetic.**

**The example relationship and all others of similar structure hold true regardless of the value, positive or negative, of the unknown by the rules of conventional arithmetic.**

**In revised algebra, the following relations hold between revised multiplication and addition (conventional)-**

**arbitrary examples**

**if-** $x = +n$

**then-** $x + x + x = (+3)(x)$

---

**if-** $x = -n$

**then-** $x + x + x = (-3)(x)$

**To determine which of the two representations of " x + x + x " is applicable, the sign of the unknown is required. Since an unknown, by definition, will not reveal its sign before solution, a fused, algebraic notation which designates both possibilities on a mutually-exclusive, contingency basis and preserves the proper parallel, one-to-one correspondence of each relation is a necessary device. This is correspondent notation.**

**In revised algebra, the following relations hold between revised multiplication and addition (conventional)-**

**arbitrary example**

$$x + x + x = (\pm 3)(\pm x)$$

**This correspondent notation representation is congruent to the two ordinary representations of the prior examples.**

**The example relationship and all others of similar structure holds true regardless of the value, positive or negative, of the unknown by the rules of revised arithmetic.**

**correspondent notation- various forms**

**revised multiplication**

$$\pm a \times \pm b = +a \times +b \ \underline{\text{or}} \ -a \times -b$$

**same signs (a & b)**

---

$$\pm a \times \mp b = +a \times -b \ \underline{\text{or}} \ -a \times +b$$

**opposite signs (a & b)**

---

**revised involution**

$$\pm a^{\pm b} = +a^{+b} \ \underline{\text{or}} \ -a^{-b}$$

**same signs (a & b)**

---

$$\pm a^{\mp b} = +a^{-b} \ \underline{\text{or}} \ -a^{+b}$$

**opposite signs (a & b)**

---

**In correspondent notation, a one-to-one correspondence between <u>either</u> the upper <u>or</u> the lower signs within each contingency sign pair must be maintained. Consequently, wherever the sign of one of the two unknowns (a & b) is given, the sign of the other unknown is automatically determined. Incidentally, where the signs of the two unknowns (a & b) are the same, they are unconditionally commutative in revised multiplication.**

**missing variable formulae**

---

**addition**

**universal formulae**

$$a + b = c$$

$$c + \wedge b = a$$

$$c + \wedge a = b$$

---

**revised multiplication**

**same signs formulae (a & b)**

**conditions:** $a \neq 0$
$b \neq 0$

---

$$a \times b = c$$

$$c \times \vee b = a$$

$$c \times \vee a = b$$

---

**If a multiplier (b) of the opposite sign of the multiplicand (a) is given, the unknown identical multiplier of the same sign can be determined by opposition for use in the set of missing variable formulae for the same signs.**

**identical multipliers formulae**

$$-a \times +b = -a \times \wedge +b$$

$$+a \times -b = +a \times \wedge -b$$

---

**revised involution**

**same signs formulae (a & b)**

**conditions:** $a \neq 0$
$b \neq 0$

---

$$a^{b} = c$$

$$c^{\vee b} = a$$

$$\log c \times \vee(\log a) = b$$

---

**If an exponent (b) of the opposite sign of the base (a) is given, the unknown identical exponent of the same sign can be determined by opposition for use in the set of missing variable formulae for the same signs.**

**identical exponents formulae**

$$-a^{+b} = -a^{\wedge +b}$$

$$+a^{-b} = +a^{\wedge -b}$$

---

| | | |
|---|---|---|
| **missing variable formulae** | | |
| **revised arithmetic demonstrations** | | |
| **addition-**<br><br>**universal formulae** | | |
| **a + b = c** | **c + ∧b = a** | **c + ∧a = b** |
| **+3 + +4 = +7** | **+7 + ∧+4 =**<br><br>**+7 + –4 = +3** | **+7 + ∧+3 =**<br><br>**+7 + –3 = +4** |
| **+2 + –3 = –1** | **–1 + ∧–3 =**<br><br>**–1 + +3 = +2** | **–1 + ∧+2 =**<br><br>**–1 + –2 = –3** |
| **–4 + 0 = –4** | **–4 + ∧0 =**<br><br>**–4 + 0 = –4** | **–4 + ∧–4 =**<br><br>**–4 + +4 = 0** |
| | | |

| | | |
|---|---|---|
| **revised multiplication**<br><br>**same signs formulae**<br><br>**a & b**<br><br>**conditions:**<br><br>**a ≠ 0**<br>**b ≠ 0** | | |
| **a x b = c** | **c x ∨b = a** | **c x ∨a = b** |
| **+8 x +0.5 = +4** | **+4 x ∨+0.5 =**<br><br>**+4 x +2 = +8** | **+4 x ∨+8 =**<br><br>**+4 x +0.125 = +0.5** |
| **–2 x –5 = –10** | **–10 x ∨–5 =**<br><br>**–10 x –0.2 = –2** | **–10 x ∨–2 =**<br><br>**–10 x –0.5 = –5** |

| | | |
|---|---|---|
| **revised involution**<br><br>**same signs formulae**<br><br>**a & b**<br><br>**conditions:**<br><br>$a \neq 0$<br>$b \neq 0$ | | |
| $a^{b} = c$ | $c^{\vee b} = a$ | $\log c \times \vee(\log a) = b$ |
| $-8^{-0.\overline{3}} = -2$ | $-2^{\vee -0.\overline{3}} = -8$<br><br>$-2^{-3} = -8$ | $(\log -2) \times \vee(\log -8) =$<br><br>$-0.30103 \times \vee -0.90309 =$<br><br>$-0.30103 \times -1.10731 = -0.\overline{3}$ |
| $+3^{+2} = +9$ | $+9^{\vee +2} = +3$<br><br>$+9^{+0.5} = +3$ | $(\log +9) \times \vee(\log +3) =$<br><br>$+0.95424 \times \vee +0.47712 =$<br><br>$+0.95424 \times +2.09591 = +2$ |

**number of solutions to simple equations with one unknown**

---

**addition**

$$a + b = c$$

---

**if given-** **b & c**

**then-** **"a" has <u>one</u> unique solution.**

---

**example-**

$$a = c + \bigwedge b$$

**universal formula**

$$a + +4 = -3$$

$$a + (+4 + \bigwedge +4) = -3 + \bigwedge +4$$

$$a + (+4 + -4) = -3 + -4$$

$$a + 0 = -7$$

$$a = -7$$

---

**if given-** **a & c**

**then-** **"b" has <u>one</u> unique solution.**

---

**example-**

$$b = c + \bigwedge a$$

**universal formula**

$$-7 + b = -3$$

$$(-7 + \bigwedge -7) + b = -3 + \bigwedge -7$$

$$(-7 + +7) + b = -3 + +7$$

$$0 + b = +4$$

$$b = +4$$

---

**revised multiplication**

**a x b = c**

---

**if given- b & c**

**then- "a" has <u>one</u> unique solution.**

---

**example-**

**a = c x $\vee$b**
**universal formula**

**a x +4 = –3**

**a x (+4 x $\vee$+4) = –3 x $\vee$+4**

**a x (+4 x +0.25) = –3 x +0.25**

**a x +1 = –3 x $\wedge$+0.25**

**a = –3 x –0.25**

**a = –0.75**

---

**if given-** **a & c**

**then-** **"b" has two unique solutions (a pair of identical multipliers).**

---

**example I**

**if-** **the sign for "b" must be the same as the sign of "a"**

**then-** $b = c \times \sqrt{a}$
**same signs formula**

---

$$-12 \times -b = -3$$

$$(-12 \times \sqrt{-12}) \times -b = -3 \times \sqrt{-12}$$

$$(-12 \times -0.08\overline{3}) \times -b = -3 \times -0.08\overline{3}$$

$$-1 \times -b = -0.25$$

$$-b = -0.25$$

---

**example II**

**if-** **the sign for "b" must be opposite to the sign of "a"**

**then-** $b = \wedge c \times \vee a$
**opposite signs formula**

---

$$-12 \times +b = -3$$

$$-12 \times \wedge +b = -3$$

$$-12 \times -b = -3$$

$$-b \times -12 = -3$$

$$-b \times (-12 \times \vee -12) = -3 \times \vee -12$$

$$-b \times (-12 \times -0.08\overline{3}) = -3 \times -0.08\overline{3}$$

$$-b \times -1 = -0.25$$

$$-b = -0.25$$

$$\wedge -b = \wedge -0.25$$

$$+b = +0.25$$

---

**note-**

**Where given "a" & "c", "a" & "b" are always both of the same signs and of opposite signs since "b" has two unique real number values which are identical multipliers (opposites of each other). Thus, both formulae must be used to fully determine "b".**

**revised involution**

$$a^{b} = c$$

---

**if given-** b & c

**then-** "a" has one unique solution.

---

**example-**

$$a = c^{\vee b}$$

**universal formula**

$$a^{+3} = -2$$

$$(a^{+3})^{\vee +3} = -2^{\vee +3}$$

$$(a^{+3})^{+0.\overline{3}} = -2^{+0.\overline{3}}$$

$$a = -2^{\wedge +0.\overline{3}}$$

$$a = -2^{-0.\overline{3}}$$

$$a = -1.26$$

---

**if given-** **a & c**

**then-** **"b" has two unique solutions (a pair of identical exponents).**

---

**example I**

**if-** **the sign for "b" must be the same as the sign of "a"**

**then-** **$b = \log c \times \sqrt{(\log a)}$**
**same signs formula**

---

$$-3^{-b} = -9$$

$$-b = \log -9 \times \sqrt{(\log -3)}$$

$$-b = -0.95424 \times \sqrt{-0.47712}$$

$$-b = -0.95424 \times -2.09591$$

$$-b = -2$$

---

**example II**

**if-** **the sign for "b" must be opposite to the sign of "a"**

**then-** **$b = \wedge[(\log c) \times \vee(\log a)]$**
**opposite signs formula**

---

$$-3^{+b} = -9$$

$$+b = \wedge[(\log -9) \times \vee(\log -3)]$$

$$+b = \wedge[-0.95424 \times \vee -0.47712]$$

$$+b = \wedge[-0.95424 \times -2.09591]$$

$$+b = \wedge -2$$

$$+b = +2$$

---

**note-**

**Where given "a" & "c", "a" & "b" are always both of the same signs and of opposite signs since "b" has two unique real number values which are identical exponents (opposites of each other). Thus, both formulae must be used to fully determine "b".**

## special cases involving zero

| | addition | revised multiplication | revised involution |
|---|---|---|---|
| given-<br><br>result-<br><br>relations- | a = 0<br>b = n<br><br>c = n<br>(sum)<br><br>c = b | a = 0<br>b = n<br><br>c = 0<br>(revised product)<br><br>c = a | a = 0<br>b = n<br><br>c = 0<br>(revised power)<br><br>c = a |
| given-<br><br>result-<br><br>relations- | a = n<br>b = 0<br><br>c = n<br>(sum)<br><br>c = a | a = n<br>b = 0<br><br>c = 0<br>(revised product)<br><br>c = b | a = n<br>b = 0<br><br>c = 0<br>(revised power)<br><br>c = b |
| given-<br><br>result-<br><br>relations- | a = 0<br>b = 0<br><br>c = 0<br>(sum)<br><br>c = a<br>c = b | a = 0<br>b = 0<br><br>c = 0<br>(revised product)<br><br>c = a<br>c = b | a = 0<br>b = 0<br><br>c = 0<br>(revised power)<br><br>c = a<br>c = b |

| | | | |
|---|---|---|---|
| **conversions-**<br><br>**conventional equivalencies of revised common logarithms** | | | |
| **true range of values** | **revised notation** | **conventional notation** | **comparisons** |
| **+n > +1** | **log +n** | **log +n** | **identical** |
| **+n < +1** | **log +n** | **–(log +n)** | **different** |
| **–n > –1** | **log –n** | **log +n** | **different** |
| **–n < –1** | **log –n** | **–(log +n)** | **different** |

---

**Note: Since logarithms of negative real numbers cannot exist in conventional notation, conversions to revised notation are only possible by also substituting a false range of values to trick the conventional algorithm into working properly and giving the correct answer.**

---

**For "log –n" with a true range of values of "–n > –1" under revised notation, the algorithm requires "log +n" with a false range of values of "+n < +1" under conventional notation to work.**

**For "log –n" with a true range of values of "–n < –1" under revised notation, the algorithm requires "–(log +n)" with a false range of values of "+n > +1" under conventional notation to work.**

---

**common logarithms of integers**

**(logarithms of integers to base +10)**

| N | $\log_{+10} N$ |
|---|---|
| 0.0 | 0.0 |
| +1.0 | $\vee +\infty$ |
| +2.0 | +0.30103 |
| +3.0 | +0.47712 |
| +4.0 | +0.60206 |
| +5.0 | +0.69897 |
| +6.0 | +0.77815 |
| +7.0 | +0.84510 |
| +8.0 | +0.90309 |
| +9.0 | +0.95424 |
| +10.0 | +1.0 |
| +100.0 | +2.0 |
| +1000.0 | +3.0 |

**common logarithms of integers**

**(logarithms of integers to base –10)**

| N | $\log_{-10} N$ |
|---|---|
| 0.0 | 0.0 |
| –1.0 | $\vee -\infty$ |
| –2.0 | –0.30103 |
| –3.0 | –0.47712 |
| –4.0 | –0.60206 |
| –5.0 | –0.69897 |
| –6.0 | –0.77815 |
| –7.0 | –0.84510 |
| –8.0 | –0.90309 |
| –9.0 | –0.95424 |
| –10.0 | –1.0 |
| –100.0 | –2.0 |
| –1000.0 | –3.0 |

**common logarithms of reciprocals of integers**

**(logarithms of reciprocals of integers to base +0.1)**

| N | $\log_{+0.1} N$ |
|---|---|
| 0.0 | 0.0 |
| +1.0 | $\vee +\infty$ |
| +0.5 | +0.30103 |
| $+0.\overline{3}$ | +0.47712 |
| +0.25 | +0.60206 |
| +0.2 | +0.69897 |
| $+0.1\overline{6}$ | +0.77815 |
| +0.1428 | +0.84510 |
| +0.125 | +0.90309 |
| $+0.\overline{1}$ | +0.95424 |
| +0.1 | +1.0 |
| +0.01 | +2.0 |
| +0.001 | +3.0 |

**common logarithms of reciprocals of integers**

**(logarithms of reciprocals of integers to base –0.1)**

| N | $\log_{-0.1} N$ |
|---|---|
| 0.0 | 0.0 |
| –1.0 | $\vee -\infty$ |
| –0.5 | –0.30103 |
| $-0.\overline{3}$ | –0.47712 |
| –0.25 | –0.60206 |
| –0.2 | –0.69897 |
| $-0.1\overline{6}$ | –0.77815 |
| –0.1428 | –0.84510 |
| –0.125 | –0.90309 |
| $-0.\overline{1}$ | –0.95424 |
| –0.1 | –1.0 |
| –0.01 | –2.0 |
| –0.001 | –3.0 |

## reciprocals of common logarithms

| N | log N | ∨log N |
|---|---|---|
| 0.0 | 0.0 | 0.0 |
| ±1 | ∨±∞ | ±∞ |
| ±2 \| ±0.5 | ±0.30103 | ±3.32193 |
| ±3 \| ±$0.\overline{3}$ | ±0.47712 | ±2.09591 |
| ±4 \| ±0.25 | ±0.60206 | ±1.66096 |
| ±5 \| ±0.2 | ±0.69897 | ±1.43068 |
| ±6 \| ±$0.1\overline{6}$ | ±0.77815 | ±1.28510 |
| ±7 \| ±0.1428 | ±0.84510 | ±1.18329 |
| ±8 \| ±0.125 | ±0.90309 | ±1.10731 |
| ±9 \| ±$0.\overline{1}$ | ±0.95424 | ±1.04795 |
| ±10 \| ±0.1 | ±1 | ±1 |
| ±100 \| ±0.01 | ±2 | ±0.5 |
| ±1000 \| ±0.001 | ±3 | ±$0.\overline{3}$ |

**common logarithms**

**products as integers & reciprocals of integers**

---

---

**log ±4 x √log ±2 = ±2**

**log ±4 x √log ±0.5 = ±2**

**log ±0.25 x √log ±2 = ±2**

**log ±0.25 x √log ±0.5 = ±2**

---

**√log ±4 x log ±2 = ±0.5**

**√log ±4 x log ±0.5 = ±0.5**

**√log ±0.25 x log ±2 = ±0.5**

**√log ±0.25 x log ±0.5 = ±0.5**

---

**log ±8 x √log ±2 = ±3**

**log ±8 x √log ±0.5 = ±3**

**log ±0.125 x √log ±2 = ±3**

**log ±0.125 x √log ±0.5 = ±3**

---

**√log ±8 x log ±2 = ±0.$\overline{3}$**

**√log ±8 x log ±0.5 = ±0.$\overline{3}$**

**√log ±0.125 x log ±2 = ±0.$\overline{3}$**

**√log ±0.125 x log ±0.5 = ±0.$\overline{3}$**

---

---

$\log \pm 9 \times \vee\log \pm 3 = \pm 2$

$\log \pm 9 \times \vee\log \pm 0.\overline{3} = \pm 2$

$\log \pm 0.\overline{1} \times \vee\log \pm 3 = \pm 2$

$\log \pm 0.\overline{1} \times \vee\log \pm 0.\overline{3} = \pm 2$

---

$\vee\log \pm 9 \times \log \pm 3 = \pm 0.5$

$\vee\log \pm 9 \times \log \pm 0.\overline{3} = \pm 0.5$

$\vee\log \pm 0.\overline{1} \times \log \pm 3 = \pm 0.5$

$\vee\log \pm 0.\overline{1} \times \log \pm 0.\overline{3} = \pm 0.5$

---

**commentary**

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**In revised involution, the common logarithm (log) of a positive real number is applied to base +10 if the given is greater than +1 or is applied to base +0.1 if the given is lesser than +1 and greater than zero. In either case, the common logarithm which is of the same sign as the positive base (a) and not its identical exponent should be used. Where two givens are reciprocals, their common logarithms are identical positive real numbers.**

**In revised involution, the common logarithm (log) of a negative real number is applied to base –10 if the given is lesser than –1 or is applied to base –0.1 if the given is greater than –1 and lesser than zero. In either case, the common logarithm which is of the same sign as the negative base (a) and not its identical exponent should be used. Where two givens are reciprocals, their common logarithms are identical negative real numbers.**

**The tables of common logarithms for positive real numbers greater than +1 as used in conventional involution and revised involution are identical. However, their applicability to negative real numbers as negative logarithms is exemplary of revised involution exclusively. In every case, the absolute values of the positive and the negative logarithm of a positive and a negative real number with equal absolute values, respectively, are also equal.**

**The described relations are universal and as such, also hold true with natural logarithms (base ±e and ∨±e) and logarithms to any base (a) which are sets of four real numbers.**

**The revised logarithm of any given real number (N) can be accommodated by a set of four real numbers as the base (a) over separate domains which are arithmetic functions of one another. On the extended real number continuum, these sets are the four cornerpoints of an inscribed rectangle.**

**With revised logarithms, a subtle, special notation is used occasionally to indicate "whichever is appropriate" among the set of four real numbers which serve as the base (a) over separate domains.**

**In revised logarithms, where any real number may be the given, revised power (N), the set of four real numbers as the base (a) is-**

$$(+a, -a, \vee+a, \vee-a)$$

**As a convention, a signless number with an absolute value greater than +1 is used to represent each four-real-number base (a). Nonetheless, this is <u>never</u> intended to represent a single, positive real number base as an implied positive which is typical in conventional math.**

**revised logarithms**

**notation to represent base (a) as four real numbers**

---

$$\log N = \log_{10} N = \log_{(\pm 10 | \pm 0.1)} N$$

---

$$\ln N = \log_{e} N = \log_{(\pm e | \vee \pm e)} N$$

---

$$\log_{a} N = \log_{(\pm a | \vee \pm a)} N$$

---

**example**

**if-** $a = \pm 8 | \pm 0.125$

---

**then-** $\log_{a} N = \log_{(\pm 8 | \pm 0.125)} N = \log_{8} N$

---

**graph lines**
**revised binary operations**

---

**addition**

**major axes**

**x axis**

$a + b = a$

---

$a = n$

$b = 0$

---

**y axis**

$a + b = b$

---

$a = 0$

$b = n$

---

**index lines**

**zero line**

$a + b = 0$

---

$a = \wedge b$

$b = \wedge a$

---

**doubles line**

$a + b = a + a$

$a + b = b + b$

---

$a = b$

**revised multiplication**

**major axes**

**x axis**

**a x b = a**

**b = ±1**

**a = n**

**y+ axis**

**a x +b = +b**

**a = +1**

**+b = +n**

**y– axis**

**a x –b = –b**

**a = –1**

**–b = –n**

**index lines**

**zero line**

**a x b = 0**

**a = 0**

**b = n**

**+1 line**

**+a x b = +1**

---

| | |
|---|---|
| **if-** | **+a x +b = +1** |
| **then-** | **+b = ⋁+a** |
| | **+a = ⋁+b** |

---

| | |
|---|---|
| **if-** | **+a x –b = +1** |
| **then-** | **–b = ⋀⋁+a** |
| | **+a = ⋀⋁–b** |

---

**–1 line**

**–a x b = –1**

---

| | |
|---|---|
| **if-** | **–a x –b = –1** |
| **then-** | **–b = ⋁–a** |
| | **–a = ⋁–b** |

---

| | |
|---|---|
| **if-** | **–a x +b = –1** |
| **then-** | **+b = ⋀⋁–a** |
| | **–a = ⋀⋁+b** |

---

**positive squares line**

**+a x b = +a x +a**

---

**if-** **+a x +b = +a x +a**

**then-** **+b = +a**

---

**if-** **+a x –b = +a x +a**

**then-** **–b = ∧+a**

**+a = ∧–b**

---

**negative squares line**

**–a x b = –a x –a**

---

**if-** **–a x –b = –a x –a**

**then-** **–b = –a**

---

**if-** **–a x +b = –a x –a**

**then-** **+b = ∧–a**

**–a = ∧+b**

---

**revised involution**

**major axes**

**x axis**

$$a^{b} = a$$

---

$$b = \pm 1$$

$$a = n$$

---

**y+ axis**

$$a^{+b} = +b$$

---

$$a = +b^{\vee +b}$$

$$+b = +n$$

---

**y– axis**

$$a^{-b} = -b$$

---

$$a = -b^{\vee -b}$$

$$-b = -n$$

---

**index lines**

**zero line**

$$a^{b} = 0$$

$$a = 0$$

$$b = n$$

**+1 line**

$$+a^{b} = +1$$

$$+a = +1$$

$$b = n$$

**–1 line**

$$-a^{b} = -1$$

$$-a = -1$$

$$b = n$$

**positive hyper-squares line**

$$\overset{b}{+a} = \overset{+a}{+a}$$

---

if- $\overset{+b}{+a} = \overset{+a}{+a}$

then- $+b = +a$

---

if- $\overset{-b}{+a} = \overset{+a}{+a}$

then- $-b = \wedge{+a}$

$+a = \wedge{-b}$

---

**negative hyper-squares line**

$$\overset{b}{-a} = \overset{-a}{-a}$$

---

if- $\overset{-b}{-a} = \overset{-a}{-a}$

then- $-b = -a$

---

if- $\overset{+b}{-a} = \overset{-a}{-a}$

then- $+b = \wedge{-a}$

$-a = \wedge{+b}$

---

**positive hyper-square roots line**

$$+a^{b} = +a^{\vee +a}$$

---

if- $+a^{-b} = +a^{\vee +a}$

then- $-b = \wedge\vee +a$

$+a = \wedge\vee -b$

---

if- $+a^{+b} = +a^{\vee +a}$

then- $+b = \vee +a$

$+a = \vee +b$

---

**negative hyper-square roots line**

$$-a^{b} = -a^{\vee -a}$$

---

if- $-a^{+b} = -a^{\vee -a}$

then- $+b = \wedge\vee -a$

$-a = \wedge\vee +b$

---

if- $-a^{-b} = -a^{\vee -a}$

then- $-b = \vee -a$

$-a = \vee -b$

---

**graph lines**

**exact values for curved major axes and curved index lines**
**(selected points plotted)**

---

**addition**

**Note- All major axes and index lines are graphed as straight lines in this revised binary operation.**

**revised multiplication**

**+1 line**

---

**+8 x +0.125 = +1**

**+4 x +0.25 = +1**

**+2 x +0.5 = +1**

**+1 x +1 = +1**

**+0.875 x +1.1428 ... = +1**

**+0.75 x +1.$\overline{3}$ = +1**

**+0.625 x +1.6 = +1**

**+0.5 x +2 = +1**

**+0.4 x +2.5 = +1**

**+0.$\overline{3}$ x +3 = +1**

**+0.25 x +4 = +1**

**+0.2 x +5 = +1**

**+0.1$\overline{6}$ x +6 = +1**

**+0.1428 ... x +7 = +1**

**+0.125 x +8 = +1**

**quad IV**
**straight line**

---

**quad I**
**curve**

**–1 line**

**–8 x –0.125 = –1**

**–4 x –0.25 = –1**

**–2 x –0.5 = –1**

**–1 x –1 = –1**

**–0.875 x –1.1428 ... = –1**

**–0.75 x $-1.\overline{3}$ = –1**

**–0.625 x –1.6 = –1**

**–0.5 x –2 = –1**

**–0.4 x –2.5 = –1**

**$-0.\overline{3}$ x –3 = –1**

**–0.25 x –4 = –1**

**–0.2 x –5 = –1**

**$-0.1\overline{6}$ x –6 = –1**

**–0.1428 ... x –7 = –1**

**–0.125 x –8 = –1**

**quad III**
**straight line**

**quad II**
**curve**

**positive squares line**

---

**+8 x +8 = +64**

**+4 x +4 = +16**

**+2 x +2 = +4**

**+1 x +1 = +1**

**+0.875 x +0.875 = +0.765625**

**+0.75 x +0.75 = +0.5625**

**+0.625 x +0.625 = +0.390625**

**+0.5 x +0.5 = +0.25**

**+0.4 x +0.4 = +0.16**

**$+0.\overline{3}$ x $+0.\overline{3}$ = $+0.\overline{1}$**

**+0.25 x +0.25 = +0.0625**

**+0.2 x +0.2 = +0.04**

**$+0.1\overline{6}$ x $+0.1\overline{6}$ = $+0.02\overline{7}$**

**+0.1428 ... x +0.1428 ... = +0.0204 ...**

**+0.125 x +0.125 = +0.015625**

---

**quad I**
**straight line**

---

**quad IV**
**curve**

**negative squares line**

---

**–8 x –8 = –64**

**–4 x –4 = –16**

**–2 x –2 = –4**

**–1 x –1 = –1**

**–0.875 x –0.875 = –0.765625**

**–0.75 x –0.75 = –0.5625**

**–0.625 x –0.625 = –0.390625**

**–0.5 x –0.5 = –0.25**

**–0.4 x –0.4 = –0.16**

**$-0.\overline{3} \times -0.\overline{3} = -0.\overline{1}$**

**–0.25 x –0.25 = –0.0625**

**–0.2 x –0.2 = –0.04**

**$-0.1\overline{6} \times -0.1\overline{6} = -0.02\overline{7}$**

**–0.1428 ... x –0.1428 ... = –0.0204 ...**

**–0.125 x –0.125 = –0.015625**

---

**quad II**
**straight line**

---

**quad III**
**curve**

**revised involution**

**y+ axis**

**_____**

**+4**
**+1.4142 ... = +4**

**+3**
**+1.4422 ... = +3**

**+2.75**
**+1.4446 ... = +2.75**

**+e**
**+1.444667 ... = +e** **(+e = +2.71828 ...)**

**+2.7**
**+1.4446 ... = +2.7**

**+2.5**
**+1.4426 ... = +2.5**

**+2**
**+1.4142 ... = +2**

**+1.5**
**+1.3103 ... = +1.5**

**+1.25**
**+1.1954 ... = +1.25**

**quadrant I**
**curve**
**_____**

**+1**
**+1 = +1**

$+1^{+1} = +1$

$+0.8584\ldots^{+0.875} = +0.875$

$+0.6814\ldots^{+0.75} = +0.75$

$+0.4714\ldots^{+0.625} = +0.625$

$+0.25^{+0.5} = +0.5$

$+0.1011^{+0.4} = +0.4$

$+0.\overline{037}^{+0.\overline{3}} = +0.\overline{3}$

$+0.018\ldots^{+0.3} = +0.3$

$+0.00390625^{+0.25} = +0.25$

quadrant IV curve

**y– axis**

$$-1.4142\ldots^{-4} = -4$$

$$-1.4422\ldots^{-3} = -3$$

$$-1.4446\ldots^{-2.75} = -2.75$$

$$-1.444667\ldots^{-e} = -e \qquad (-e = -2.71828\ldots)$$

$$-1.4446\ldots^{-2.7} = -2.7$$

$$-1.4426\ldots^{-2.5} = -2.5$$

$$-1.4142\ldots^{-2} = -2$$

$$-1.3103\ldots^{-1.5} = -1.5$$

$$-1.1954\ldots^{-1.25} = -1.25$$

$$-1^{-1} = -1$$

**quadrant II curve**

$-1^{-1} = -1$

$-0.8584\ldots^{-0.875} = -0.875$

$-0.6814\ldots^{-0.75} = -0.75$

$-0.4714\ldots^{-0.625} = -0.625$

$-0.25^{-0.5} = -0.5$

$-0.1011^{-0.4} = -0.4$

$-0.\overline{037}^{-0.\overline{3}} = -0.\overline{3}$

$-0.018\ldots^{-0.3} = -0.3$

$-0.00390625^{-0.25} = -0.25$

---

**quadrant III curve**

**positive hyper-squares line**

---

$+4^{+4} = +256$

$+3^{+3} = +27$

$+2^{+2} = +4$

$+1^{+1} = +1$

**quadrant I**
**straight line**

---

**quadrant IV**
**curve**

$+0.875^{+0.875} = +0.8897 \ldots$

$+0.75^{+0.75} = +0.8059 \ldots$

$+0.625^{+0.625} = +0.7454 \ldots$

$+0.5^{+0.5} = +0.7071 \ldots$

$+0.4^{+0.4} = +0.6931 \ldots$

$\forall{+e}^{\forall{+e}} = +0.6922 \ldots$

$+0.\overline{3}^{+0.\overline{3}} = +0.6933 \ldots$

$+0.3^{+0.3} = +0.6968 \ldots$

$+0.25^{+0.25} = +0.7071 \ldots$

---

**negative hyper-squares line**

---

$$-4^{-4} = -256$$

$$-3^{-3} = -27$$

$$-2^{-2} = -4$$

$$-1^{-1} = -1$$

**quadrant II straight line**

---

**quadrant III curve**

$$-0.875^{-0.875} = -0.8897 \ldots$$

$$-0.75^{-0.75} = -0.8059 \ldots$$

$$-0.625^{-0.625} = -0.7454 \ldots$$

$$-0.5^{-0.5} = -0.7071 \ldots$$

$$-0.4^{-0.4} = -0.6931 \ldots$$

$$\sqrt{-e}^{\sqrt{-e}} = -0.6922 \ldots$$

$$-0.\overline{3}^{-0.\overline{3}} = -0.6933 \ldots$$

$$-0.3^{-0.3} = -0.6968 \ldots$$

$$-0.25^{-0.25} = -0.7071 \ldots$$

---

**positive hyper-square roots line**

---

$\sqrt{+4}$
**+4 = +1.4142 ...**

$\sqrt{+3}$
**+3 = +1.4422 ...**

$\sqrt{+e}$
**+e = +1.444667 ...**

$\sqrt{+2}$
**+2 = +1.4142 ...**

**quadrant IV straight line**

---

$\sqrt{+1}$
**+1 = +1**

**quadrant I curve**

$\sqrt{+0.875}$
**+0.875 = +0.8584 ...**

$\sqrt{+0.75}$
**+0.75 = +0.6814 ...**

$\sqrt{+0.625}$
**+0.625 = +0.4714 ...**

$\sqrt{+0.5}$
**+0.5 = +0.25**

$\sqrt{+0.4}$
**+0.4 = +0.1011 ...**

$\sqrt{+0.\overline{3}}$
$+0.\overline{3} = +0.\overline{037}$

$\sqrt{+0.3}$
**+0.3 = +0.018 ...**

$\sqrt{+0.25}$
**+0.25 = +0.00390625**

---

**negative hyper-square roots line**

---

$$-4^{\sqrt{-4}} = -1.4142\ ...$$

$$-3^{\sqrt{-3}} = -1.4422\ ...$$

$$-e^{\sqrt{-e}} = -1.444667\ ...$$

$$-2^{\sqrt{-2}} = -1.4142\ ...$$

$$-1^{\sqrt{-1}} = -1$$

**quadrant III**
**straight line**

---

$$-0.875^{\sqrt{-0.875}} = -0.8584\ ...$$

$$-0.75^{\sqrt{-0.75}} = -0.6814\ ...$$

$$-0.625^{\sqrt{-0.625}} = -0.4714\ ...$$

$$-0.5^{\sqrt{-0.5}} = -0.25$$

$$-0.4^{\sqrt{-0.4}} = -0.1011\ ...$$

$$-0.\overline{3}^{\sqrt{-0.\overline{3}}} = -0.\overline{037}$$

$$-0.3^{\sqrt{-0.3}} = -0.018\ ...$$

$$-0.25^{\sqrt{-0.25}} = -0.00390625$$

---

**quadrant II**
**curve**

**factorial notation with revised multiplication**

---

**positive factorials**

**+n = a positive integer**

**+n! = +n x (+n + -1) x (+n + -2) x (+n + -3) ...**

**(note- the last expression must equal +1)**

---

**negative factorials**

**-n = a negative integer**

**-n! = -n x (-n + +1) x (-n + +2) x (-n + +3) ...**

**(note- the last expression must equal -1)**

---

**zero factorial**

**an arbitrary, special case**

**0! = ±1 (as correspondent notation for same signs)**

**examples-**

**0! x +3! = 0! x +6 = +1 x +6 = +6**

**0! x -3! = 0! x -6 = -1 x -6 = -6**

---

**integer factorials**

---

**zero factorial**

**0! = ±1 (as correspondent notation for same signs)**

---

**positive factorials**

**+1! = +1**

**+2! = +2 = +2 x +1**

**+3! = +6 = +3 x +2 x +1**

**+4! = +24 = +4 x +3 x +2 x +1**

**+5! = +120 = +5 x +4 x +3 x +2 x +1**

**+6! = +720 = +6 x +5 x +4 x +3 x +2 x +1**

**+7! = +5040 = +7 x +6 x +5 x +4 x +3 x +2 x +1**

**+8! = +40320 = +8 x +7 x +6 x +5 x +4 x +3 x +2 x +1**

---

**negative factorials**

**-1! = -1**

**-2! = -2 = -2 x -1**

**-3! = -6 = -3 x -2 x -1**

**-4! = -24 = -4 x -3 x -2 x -1**

**-5! = -120 = -5 x -4 x -3 x -2 x -1**

**-6! = -720 = -6 x -5 x -4 x -3 x -2 x -1**

**-7! = -5040 = -7 x -6 x -5 x -4 x -3 x -2 x -1**

**-8! = -40320 = -8 x -7 x -6 x -5 x -4 x -3 x -2 x -1**

**scientific notation**

**examples-**

**positive real numbers**

$$+5{,}000{,}000 = +5.0 \times +10^{+6}$$

$$+5{,}000 = +5.0 \times +10^{+3}$$

$$+500 = +5.0 \times +10^{+2}$$

$$+50 = +5.0 \times +10^{\pm 1} = +5 \times +10$$

$$+5.0 = +5.0$$

$$+0.5 = +5.0 \times +0.1^{\pm 1} = +5 \times +0.1$$

$$+0.05 = +5.0 \times +0.1^{+2}$$

$$+0.005 = +5.0 \times +0.1^{+3}$$

$$+0.000005 = +5.0 \times +0.1^{+6}$$

**examples-**

**negative real numbers**

$$-5{,}000{,}000 = -5.0 \times -10^{-6}$$

$$-5{,}000 = -5.0 \times -10^{-3}$$

$$-500 = -5.0 \times -10^{-2}$$

$$-50 = -5.0 \times -10^{\pm 1} = -5 \times -10$$

$$-5.0 = -5.0$$

$$-0.5 = -5.0 \times -0.1^{\pm 1} = -5 \times -0.1$$

$$-0.05 = -5.0 \times -0.1^{-2}$$

$$-0.005 = -5.0 \times -0.1^{-3}$$

$$-0.000005 = -5.0 \times -0.1^{-6}$$

**law of exponents**

---

$$a^{b-1} \times a^{b-2} = a^{[b-1 + b-2]}$$

**same signs formula**
**(a, b-1 & b-2)**

---

**laws of logarithms**

---

$$\log_a c = \log_e c \times \vee \log_e a$$

---

$$\log_a c = b$$

---

$$\log_{\vee a} \vee c = b$$

---

$$\log_{\vee a} \vee c = \log_a c$$

---

$$\wedge(\log_a c) = \wedge b$$

---

$$\log_{\wedge a} \wedge c = \wedge b$$

---

$$\log_{\wedge a} \wedge c = \wedge(\log_a c)$$

---

$$\log_{\wedge \vee a} \wedge \vee c = \log_{\wedge a} \wedge c$$

---

$$\log_c a = \vee b$$

---

$$\vee(\log_c a) = b$$

---

$$\wedge(\log_c a) = \wedge\vee b$$

---

$$\wedge\vee(\log_c a) = \wedge b$$

---

$$\log_c (a \times b) = \log_c a + \log_c b$$

**same signs only**
**(a & b)**

---

**revised involution**

**special case- two real numbers which as equivalent exponents (b) and revised powers (c) satisfy equations with a single base (a)**

---

**In the special case for revised involution in which the exponent (b) and the revised power (c) equal the same real number (n), there are two, unique real numbers which satisfy an equation to a single base (a), provided it has an absolute value greater than +1.**

---

$a^b = c$ revised involution

$b = c$ special case relation

$n = b = c$

$a^n = n$ revised involution- special case

---

$n-1 = b-1 = c-1$

$n-2 = b-2 = c-2$

---

**n-1 = real number #1 (of lesser absolute value than n-2) representing exponent #1 (b-1) and revised power #1 (c-1).**

**n-2 = real number #2 (of greater absolute value than n-1) representing exponent #2 (b-2) and revised power #2 (c-2).**

---

$a^{n-1} = n-1$

(condition: $|a| > +1$ )

$a^{n-2} = n-2$

(condition: $|a| > +1$ )

**examples-**

**using the >+1 ray of the y+ axis**

| n-1 (low value) | n-2 (high value) |
|---|---|
| $+1.1143^{+1.13} = +1.13$ | $+1.1143^{+32} = +32$ |
| $+1.1892^{+1.24} = +1.24$ | $+1.1892^{+16} = +16$ |
| $+1.2968^{+1.465} = +1.465$ | $+1.2968^{+8} = +8$ |
| $+1.348^{+1.625} = +1.625$ | $+1.348^{+6} = +6$ |
| $+1.38^{+1.77} = +1.77$ | $+1.38^{+5} = +5$ |
| $+1.414^{+2} = +2$ | $+1.414^{+4} = +4$ |
| $+1.4422^{+2.5} = +2.5$ | $+1.4422^{+3} = +3$ |
| $+1.444667^{+e} = +e$ | $+1.444667^{+e} = +e$ |

**In one special case within this special case for revised involution, where the equivalent exponent (b) and revised power (c) both equal "±e", there is only one unique real number which satisfies the equation as the single base (a).**

$$\pm a^{\pm e} = \pm e$$ **special case exception**

$$\pm a = \pm e^{\vee \pm e} = \pm 1.444667 \ldots$$

**On the graph of revised involution, "base ±e to exponent ∨±e" defines two values which are the maxima measured on the x axis for the y+ axis and minima measured on the x axis for the y– axis. Other lines perpendicular to the x axis which cut the y+ axis in quadrant I or cut the y– axis in quadrant II intersect pairs of points as secants which have the described numerical relation. The maxima and minima are special cases in which the lines perpendicular to the x axis are tangents to the y+ and y– axes, respectively, intersecting only one point.**

**There is no well-known numerical function in conventional math which gives "n-1" when "n-2" is known or vice versa. However, there is a simple formula to determine the exact base (a) for a real number (n) as an equivalent exponent (b) and revised power (c). With a very large number of calculations, pairs of unique real numbers (n-1 & n-2) with identical bases could be matched and a detailed table compiled.**

$$n^{\vee n}$$ **table generation formula**

---

| properties of the revised real number system | | | | |
|---|---|---|---|---|
| **part I**<br><br>**elements of identity, unity and elimination** | **given** | **identity element**<br>_____<br>**c = a**<br>**c = b** | **unity element**<br>_____<br>**c = ±1** | **elimination element**<br>_____<br>**c = 0** |
| **addition** | **a**<br><br>**b** | **b = 0**<br><br>**a = 0** | **- N/A -**<br><br>**- N/A -** | **b = ∧a**<br><br>**a = ∧b** |
| **revised multiplication** | **a**<br>**+a**<br>**–a**<br>**b**<br>**+b**<br>**–b** | **b = ±1**<br>**b = ±1**<br>**b = ±1**<br>**a = ±1**<br>**a = +1**<br>**a = –1** | **b = ∨a**<br>**b = ∨+a**<br>**b = ∨–a**<br>**a = ∨±b**<br>**a = ∨+b**<br>**a = ∨–b** | **b = 0**<br>**b = 0**<br>**b = 0**<br>**a = 0**<br>**a = 0**<br>**a = 0** |
| **revised involution** | **a**<br>**+a**<br>**–a**<br><br>**b** | **b = ±1**<br>**b = ±1**<br>**b = ±1**<br>∨**b**<br>**a = b** | **b = ∨±∞**<br>**b = ∨+∞**<br>**b = ∨–∞**<br><br>**a = ±1** | **b = 0**<br>**b = 0**<br>**b = 0**<br><br>**a = 0** |

**part I**

**full equations**

---

**addition**

**identity element**

| | |
|---|---|
| **given- "a"** | $a + 0 = a$ |
| **given- "b"** | $0 + b = b$ |

**unity element**

**- none -**

**elimination element**

| | |
|---|---|
| **given- "a"** | $a + \wedge a = 0$ |
| **given- "b"** | $\wedge b + b = 0$ |

**revised multiplication**

**identity element**

| | |
|---|---|
| **given- "a"** | $a \times \pm 1 = a$ |
| **given- "b"** | $\pm 1 \times \pm b = \pm b$ |
| **given- "+b"** | $+1 \times +b = +b$ |
| **given- "–b"** | $-1 \times -b = -b$ |

**unity element**

| | |
|---|---|
| **given- "a"** | $\pm a \times \vee\pm a = \pm 1$ |
| **given- "+a"** | $+a \times \vee+a = +1$ |
| **given- "–a"** | $-a \times \vee-a = -1$ |
| **given- "b"** | $\vee\pm b \times \pm b = \pm 1$ |
| **given- "+b"** | $\vee+b \times +b = +1$ |
| **given- "–b"** | $\vee-b \times -b = -1$ |

**elimination element**

| | |
|---|---|
| **given- "a"** | $a \times 0 = 0$ |
| **given- "b"** | $0 \times b = 0$ |

**revised involution**

**identity element**

| | |
|---|---|
| **given- "a"** | $a^{\pm 1} = a$ |
| **given- "b"** | $(b^{\vee b})^{b} = b$ |

**unity element**

**given- "a"** $\pm a^{\vee\pm\infty} \ddot{=} \pm 1$

---

**given- "+a"** $+a^{\vee+\infty} \ddot{=} +1$

**given- "–a"** $-a^{\vee-\infty} \ddot{=} -1$

---

**given- "b"** $\pm 1^{b} = \pm 1$

**elimination element**

**given- "a"** $a = 0^{0}$

**given- "b"** $0 = 0^{b}$

---

**elements of identity, unity and elimination**

**implicit theorems**

---

**unity element**

**revised involution**

**if-** $a^{b} = \pm 1$

**then-** $a = \pm 1$

---

**elimination element**

**revised multiplication**

**if-** $a \times b = 0$

**then-** $a = 0$ **and/or** $b = 0$

**revised involution**

**if-** $a^{b} = 0$

**then-** $a = 0$ **and/or** $b = 0$

---

**part II**

**properties of arrangement**

---

**closure**

| | |
|---|---|
| **addition** | $a + b \in \mathbf{R}$ |
| **revised multiplication** | $a \times b \in \mathbf{R}$ |
| **revised involution** | $a^{b} \in \mathbf{R}$ |

**commutative**

| | |
|---|---|
| **addition** | $a + b = b + a$ |
| **revised multiplication** | $a \times b = b \times a$ |

**note- "a" & "b" must be of the same sign.**
**"b" is convertible to the same sign as "a".**

**distributive**

**revised multiplication over addition**

**multiplier arrangement** $(a + b) \times c = ac + bc$

**note- "a" & "b" must be of the same sign.**
**"a" & "b" are not convertible.**

---

**multiplicand arrangement** $c(a + b) = ca + cb$

**note- "a", "b" & "c" must be of the same sign.**
**"a", "b" & "c" are not convertible.**

---

**revised involution over revised multiplication**

**exponent arrangement**

$$[a \times b]^{c} = a^{c} \times b^{c}$$

**note- "a" & "b" must be of the same sign.**
**"b" is convertible to the same sign as "a".**

---

**part III**

**properties of equality**

---

**reflexive**

$$a = a$$

**symmetric**

if- $a = b$

then- $b = a$

**transitive**

if- $a = b$
$b = c$

then- $a = c$

**revised binary operations**

**addition**

if- $a = b$

then- $a + c = b + c$
$c + a = c + b$

**revised multiplication**

if- $a = b$

then- $a \times c = b \times c$
$c \times a = c \times b$

**revised involution**

if- $a = b$

then- $a^{c} = b^{c}$

$c^{a} = c^{b}$

## cancellation

### addition

if- $a + c = b + c$

then- $a = b$

---

if- $c + a = c + b$

then- $a = b$

---

### revised multiplication

if- $a \times c = b \times c$

then- $a = b$

---

if- $c \times a = c \times b$

then- $a = b$

---

note- "a" & "b" must be of the same sign.
"a" is convertible to the same sign as "b".
"b" is convertible to the same sign as "a".

### revised involution

if- $a^c = b^c$

then- $a = b$

---

if- $c^a = c^b$

then- $a = b$

---

note- "a" & "b" must be of the same sign.
"a" is convertible to the same sign as "b".
"b" is convertible to the same sign as "a".

**substitution**

**addition**

**if-** $a = b$ **and** $c = d$

**then-** $a + c = b + d$
$c + a = d + b$

**revised multiplication**

**if-** $a = b$ **and** $c = d$

**then-** $a \times c = b \times d$
$c \times a = d \times b$

**revised involution**

**if-** $a = b$ **and** $c = d$

**then-** $a^{c} = b^{d}$

$c^{a} = d^{b}$

---

**part IV**

**laws of opposition and reciprocation**

---

$\wedge +n = -n$

$\wedge -n = +n$

$\wedge \pm n = \mp n$

$\wedge \mp n = \pm n$

---

$n + \wedge n = 0$

$\wedge n + n = 0$

---

$+n \times \vee +n = +1$

$\vee +n \times +n = +1$

$-n \times \vee -n = -1$

$\vee -n \times -n = -1$

$\pm n \times \vee \pm n = \pm 1$

$\vee \pm n \times \pm n = \pm 1$

$\mp n \times \vee \mp n = \mp 1$

$\vee \mp n \times \mp n = \mp 1$

---

$a \times +b = a \times \wedge +b$

$a \times -b = a \times \wedge -b$

$a \times b = a \times \wedge b$

---

$a \times \wedge +b = a \times +b$

$a \times \wedge -b = a \times -b$

$a \times \wedge b = a \times b$

---

$a^{+b} = a^{\wedge +b}$

$a^{-b} = a^{\wedge -b}$

$a^{b} = a^{\wedge b}$

---

$a^{\wedge +b} = a^{+b}$

$a^{\wedge -b} = a^{-b}$

$a^{\wedge b} = a^{b}$

---

$\bigwedge(a + b) = \bigwedge a + \bigwedge b$ **important**

---

$\bigwedge(a \times b) = \bigwedge a \times \bigwedge b$ **important**

$\bigwedge(a \times b) = \bigwedge a \times b$

$\bigvee(a \times b) = \bigvee a \times \bigvee b$

$\bigvee(a \times b) = \bigvee a \times \bigwedge\bigvee b$

$\bigwedge\bigvee(a \times b) = \bigwedge\bigvee a \times \bigwedge\bigvee b$

$\bigwedge\bigvee(a \times b) = \bigwedge\bigvee a \times \bigvee b$

---

$\bigwedge[a^{b}] = \bigwedge a^{\bigwedge b}$ **important**

$\bigwedge[a^{b}] = \bigwedge a^{b}$

$\bigvee[a^{b}] = \bigvee a^{b}$

$\bigvee[a^{b}] = \bigvee a^{\bigwedge b}$

$\bigwedge\bigvee[a^{b}] = \bigwedge\bigvee a^{\bigwedge b}$

$\bigwedge\bigvee[a^{b}] = \bigwedge\bigvee a^{b}$

---

**laws of opposition and reciprocation**

**important derivative laws**

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**distributive property-**

**opposition over addition**

---

**universal law**

$\wedge(a + b) = \wedge a + \wedge b$

**derivative laws (w/signed real numbers)**

$\wedge(+a + +b) = \wedge +a + \wedge +b = -a + -b$

$\wedge(+a + -b) = \wedge +a + \wedge -b = -a + +b$

$\wedge(-a + +b) = \wedge -a + \wedge +b = +a + -b$

$\wedge(-a + -b) = \wedge -a + \wedge -b = +a + +b$

---

**summary-**

**There are four possible combinations of signed real numbers as summands (a & b) with four corresponding, unique sums (c). Two pairs of combinations of signed summands (a & b), corresponding to or representing two pairs of sums (c), are defined as opposites.**

---

**distributive property-**

**opposition over revised multiplication**

---

**universal law**

$\wedge(a \times b) = \wedge a \times \wedge b$

**derivative laws (w/signed real numbers)**

$\wedge(+a \times +b) = \wedge +a \times \wedge +b = -a \times -b$

$\wedge(+a \times -b) = \wedge +a \times \wedge -b = -a \times +b$

$\wedge(-a \times +b) = \wedge -a \times \wedge +b = +a \times -b$

$\wedge(-a \times -b) = \wedge -a \times \wedge -b = +a \times +b$

---

**summary-**

**There are four possible combinations of signed real numbers as factors (a & b) with four corresponding, unique revised products (c). Two pairs of combinations of signed factors (a & b), corresponding to or representing two pairs of revised products (c), are defined as opposites.**

---

**distributive property-**

**opposition over revised involution**

---

**universal law**

$$\wedge[a^{b}] = \wedge a^{\wedge b}$$

**derivative laws (w/signed real numbers)**

$$\wedge[+a^{+b}] = \wedge +a^{\wedge +b} = -a^{-b}$$

$$\wedge[+a^{-b}] = \wedge +a^{\wedge -b} = -a^{+b}$$

$$\wedge[-a^{+b}] = \wedge -a^{\wedge +b} = +a^{-b}$$

$$\wedge[-a^{-b}] = \wedge -a^{\wedge -b} = +a^{+b}$$

---

**summary-**

**There are four possible combinations of signed real numbers as a base (a) and an exponent (b) with four corresponding, unique revised powers (c). Two pairs of combinations of signed base-exponent pairs (a & b), corresponding to or representing two pairs of revised powers (c), are defined as opposites.**

---

**revised binary operations description**

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**addition**

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**Compared to conventional arithmetic, there are absolutely no changes instituted in the first conventional binary operation "(conventional) addition" under revised arithmetic. However, the continued use of its inverse operation "conventional subtraction", being formally redundant and unnecessary to revised and conventional arithmetic, is eliminated.**

**This binary operation is graphed using the rectangular coordinate system in the familiar manner. The x and y axes are ordinary real number lines, each consisting of every positive and negative real number as well as zero, which intersect perpendicularly at each other's midpoint of zero and form an origin at (0, 0).**

**In the x y axes plane, there are two index lines: the zero line and the doubles line or revised multiplication line.**

**The zero line consists of all sums (c) that equal zero for every pair of summands (a & b) that are opposites.**

**The doubles line or revised multiplication line consists of all sums (c) that equal "a + a" for each pair of summands (a & b) that are identical. This algorithm of repeated addition is the basis of revised multiplication.**

**Addition has an identity element of "zero" for givens of "a" or "b", "no unity elements" and elimination elements of "$\wedge$a" for the given of "a" and "$\wedge$b" for the given of "b".**

**Addition has closure within the set of real numbers. The associative and commutative properties apply to addition unconditionally. The cancellation property applies to addition as a derivative property of additive elimination (i.e., the addition of the elimination element which in addition is the opposite). Moreover, since the commutative property applies unconditionally, the cancellation property also applies unconditionally.**

**Since the commutative property applies conditionally in revised multiplication, the distributive property for revised multiplication over addition has two distinct expressions that also apply conditionally and are not convertible otherwise.**

**All of the applicable properties assembled necessitate the set of real numbers under addition to be a commutative group. Furthermore, the set of real numbers under addition and revised multiplication satisfies all of the properties of a field. Notwithstanding, since the distributive property for revised multiplication over addition by both expressions applies conditionally, the properties of a field are satisfied conditionally, assuming one does not slightly modify the definition of a field under revised math to accommodate this difference.**

**Regardless of the signs of the summands (a & b), one set of missing variable formulae is universal for "a", "b" & "c". Where the augend (a) has a domain of the complete set of real numbers and the addend (b) has a range of the complete set of real numbers, the sum (c) has a scope of the complete set of real numbers. The summand (a or b) that has the greatest absolute value determines the sign of the sum (c).**

---

**revised multiplication**

**A sharp departure from conventional arithmetic occurs in the second, revised binary operation "revised multiplication". The separate use of an inverse operation "revised division" is unnecessary and useless for revised arithmetic so it is never introduced. Comparatively, the revised products of positive and/or negative factors (a & b) differ by signs from conventional products in ½ of the cases although the absolute values remain identical. Consequently, the distinction between conventional multiplication and revised multiplication must always be made.**

**In revised multiplication, the rectangular coordinate system is used in a markedly different scheme from as in conventional multiplication.**
**As in conventional multiplication, the x axis is an ordinary real number line consisting of every positive and negative real number as well as zero. However, there are two y axes: the y+ axis and the y– axis. Each is an exclusively positive or negative real number line with +1 or –1, respectively, as midpoints of each axis and x-intercepts. The origin of the x y+ axes is (+1,+1). The origin of the x y– axes is (–1, –1).**

**The y+ axis consists of every positive real number with each pair of points equidistant from the +1 midpoint existing as reciprocals on the ">+1 ray" and the "<+1 ray". The y– axis consists of every negative real number with each pair of points equidistant from the –1 midpoint existing as reciprocals on the "<–1 ray" and the ">–1 ray". Incidentally, there is no accommodation for zero on either the y+ or y– axes. Instead, a "y point" of zero co-exists (invisibly) at the same location as the "x point" of zero on the x axis. Geometrically, the y+ and y– axes are parallel to each other and both are perpendicular to the x axis. On the y+ and y– axes, any pair of points intersected by a line parallel to the x axis are identical multipliers (b).**

**Numerically, pairs of identical multipliers (b) exist in the relationship prescribed by the following formula as opposites-**

$$a \times b = a \times \wedge b$$

**In the x y± axes plane, there are five index lines: the zero line, the +1 line, the –1 line, the positive squares line or positive revised involution line, the negative squares line or negative revised involution line.**

**The zero line consists of all revised products (c) which equal zero for the multiplicand (a) where it equals zero and the multiplier (b) where it is every real number.**

**The +1 line consists of all revised products (c) which equal "+1" for every positive multiplicand (+a) and the multiplier (b) as the reciprocal of "+a".**

**The –1 line consists of all revised products (c) which equal "–1" for every negative multiplicand (–a) and the multiplier (b) as the reciprocal of "–a".**

**The positive squares line or positive revised involution line consists of all revised products (c) which equal "+a x +a" for every positive multiplicand (+a) and the multiplier (b) as either its equal "+a" or the identical multiplier of its equal "+a".**

**The negative squares line/negative revised involution line consists of all revised products (c) which equal "–a x –a" for every negative multiplicand (–a) and the multiplier (b) as either its equal "–a" or the identical multiplier of its equal "–a".**

**Together, these positive and negative algorithms of repeated revised multiplication are the basis of revised involution.**

**Revised multiplication has identity elements of "±1" for the given of "a", "+1" for the given of "+b" and "–1" for the given of "–b".**

**Revised multiplication has unity elements of "∨+a" for the given of "+a", "∨–a" for the given of "–a", "∨+b" for the given of "+b" and "∨–b" for the given of "–b".**

**Revised multiplication has the elimination element of "zero" for givens of "a" or "b".**

**Revised multiplication has closure within the set of real numbers.**
**The associative property applies to revised multiplication unconditionally.**

**The commutative property applies if both factors (a & b) are of the same sign, either originally or through conversion of identical multipliers (b). Potentially, this is in every case. The commutative property does not apply if the two factors (a & b) are of opposite signs until/unless they are converted to the same sign by providing the unknown, identical multiplier (b).**

**The cancellation property applies to revised multiplication as a derivative property of multiplicative unity (i.e., the revised multiplication of the unity element which in revised multiplication is the reciprocal). Since the commutative property applies conditionally in revised multiplication, the cancellation property has two distinct expressions; one of which applies unconditionally and one of which applies conditionally but is convertible.**

**Since the commutative property applies conditionally in revised multiplication, the distributive property for revised multiplication over addition has two distinct expressions that also apply conditionally and are not convertible otherwise.**

**The distributive property for revised involution over revised multiplication applies if the two factors (a & b) involved are of the same sign, either originally or through conversion of identical multipliers (b).**
**Potentially, this is in every case. The distributive property for revised involution over revised multiplication does not apply if the two factors (a & b) involved are of opposite signs until/unless they are converted to the same sign by providing the unknown, identical multiplier (b).**

**All of the applicable properties assembled necessitate the set of real numbers under revised multiplication to be a commutative group. Furthermore, the set of real numbers under revised multiplication and addition satisfies all of the properties of a field. Notwithstanding, since the distributive property for revised multiplication over addition by both expressions applies conditionally, the properties of a field are satisfied conditionally, assuming one does not slightly modify the definition of a field under revised math to accommodate this difference.**

**The signs of both factors (a & b) must be the same, either originally or through conversion of identical multipliers (b), for one set of missing variable formulae to be universal for "a", "b" & "c".**

**Where the multiplicand (a) has a domain of the complete set of real numbers and the multiplier (b) has a range of the complete set of real numbers, the revised product (c) has a scope of ½ of the set of real numbers- positive <u>or</u> negative- as the sign of the multiplicand (a) likewise determines the sign of the revised product (c).**

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**revised involution**

**The third, revised binary operation "revised involution" is built upon and incorporates revised multiplication. The separate use of an inverse operation "revised evolution" is unnecessary for revised arithmetic so it is never introduced. Comparatively, the revised powers of base-exponent pairs (a & b) that are positive and/or negative vary somewhat from conventional powers in ¾ of the cases. Consequently, the distinction between conventional involution and revised involution must always be made.**

**In revised involution, the rectangular coordinate system is used in a similar manner as in revised multiplication. The x axis is an ordinary real number line which consists of every positive and negative real number as well as zero. There are two y axes: the y+ axis and the y– axis.**

**Each is an exclusively positive or negative real number line with +1 or –1, respectively, as midpoints of each axis and x-intercepts. The origin of the x y+ axes is (+1,+1). The origin of the x y– axes is (–1, –1).**

**The y+ axis consists of every positive real number with each pair of points equidistant from the +1 midpoint existing as reciprocals on the ">+1 ray" and the "<+1 ray". The y– axis consists of every negative real number with each pair of points equidistant from the –1 midpoint existing as reciprocals on the "<–1 ray" and the ">–1 ray". Incidentally, there is no accommodation for zero on either the y+ or y– axes. Instead, a "y point" of zero co-exists (invisibly) at the same location as the "x point" of zero on the x axis. Geometrically, the y+ and y– axes are irregularly parallel to each other and both are irregularly perpendicular to the x axis. On the y+ and y– axes, any pair of points intersecting a line parallel to the x axis are identical exponents (b).**

**Numerically, pairs of identical exponents (b) exist in the relationship prescribed by the following formula as opposites-**

$$a^{b} = a^{\wedge b}$$

**Although they are prescribed by different formulae, pairs of identical exponents are numerically equivalent to pairs of identical multipliers in every case, being opposites of each other as "b".**

**Numerically, the y+ and y– axes of revised involution are identical in every respect to the y+ and y– axes, respectively, of revised multiplication, both with respect to the x axis and each other. However, their geometric relationships vary slightly. The y+ and y– axes are not perfectly parallel to each other nor do they intersect. Moreover, relative to the x axis, neither the y+ and y– axes are perfectly perpendicular. This is possible because unlike all other axes encountered, the y+ and y– axes of revised involution are not straight lines.**

---

**y+ axis**

**The maxima measured on the x axis of a line perpendicular to it and intersecting the y+ axis is precisely "base +e to exponent ∨+e" (+1.444667 ...). It is reached at a singular, corresponding y+ axis value of precisely "+e" (+2.71828 ...) in quadrant I. All x axis values greater than "+1" and lesser than "base +e to exponent ∨+e" (+1.444667 ...) have two corresponding y+ axis values in quadrant I. All x axis values lesser than "+1" and greater than "zero" have one corresponding y+ axis value in quadrant IV.**

---

**y– axis**

**The minima measured on the x axis of a line perpendicular to it and intersecting the y– axis is precisely "base –e to exponent ∨–e" (–1.444667 ...). It is reached at a singular, corresponding y– axis value of precisely "–e" (–2.71828 ...) in quadrant II. All x axis values lesser than "–1" and greater than "base –e to exponent ∨–e" (–1.444667 ...) have two corresponding y– axis values in quadrant II. All x axis values greater than "–1" and lesser than "zero" have one corresponding y– axis value in quadrant III.**

---

**In the x y± axes plane, there are seven index lines: the zero line, the +1 line, the –1 line, the positive hyper-squares line or positive revised hyper-involution line, the negative hyper-squares line or negative revised hyper-involution line, the positive hyper-square roots line and the negative hyper-square roots line.**

**The zero line consists of all revised powers (c) which equal zero for the base (a) where it equals zero and the exponent (b) where it is every real number.**

**The +1 line consists of all revised powers (c) which equal "+1" for the base (a) where it equals "+1" and the exponent (b) where it is every real number.**

**The –1 line consists of all revised powers (c) which equal "–1" for the base (a) where it equals "–1" and the exponent (b) where it is every real number.**

**The positive hyper-squares line or positive revised hyper-involution line consists of all revised powers (c) that equal "base +a to exponent +a" for every positive base (+a) and the exponent (b) as either its equal "+a" or the identical exponent of its equal "+a".**

**The negative hyper-squares line or negative revised hyper-involution line consists of all revised powers (c) which equal "base –a to exponent –a" for every negative base (–a) and the exponent (b) as either its equal "–a" or the identical exponent of its equal "–a".**

**Together, these positive and negative algorithms of repeated revised involution are the basis of "revised hyper-involution", the fourth, revised binary operation.**

**The properties of revised hyper-involution are not elaborated in detail within this work because they are only slightly important to it. Theoretically, any number of revised binary operations can be built up with each higher one incorporating the properties of its predecessor. Also, the terms "hyper-square" and "hyper-square root" are derived from a relation within the fourth, revised binary operation.**

**The positive hyper-square roots line consists of all revised powers (c) which equal "base +a to exponent $\vee$+a" for every positive base (+a) and the exponent (b) as the reciprocal of "+a". This is also the positive self-root function.**

**The negative hyper-square roots line consists of all revised powers (c) which equal "base –a to exponent $\vee$–a" for every negative base (–a) and the exponent (b) as the reciprocal of "–a". This is also the negative self-root function.**

**Revised involution has identity elements of "$\pm$1" for the given of "a" and "base b to exponent $\vee$b" for the given of "b".**

**Revised involution has unity elements of "±1" for the given of "b". Strictly speaking, there are no unity elements for the given of "a" which equal a revised power (c) of exactly "+1" or "–1". Nonetheless, for the given of "+a", "$\vee+\infty$" yields a revised power (c) which infinitely approaches "+1". If "+a > +1", this approach is from the greater than side of "+1", also known as from the right. If "+a < +1", this approach is from the lesser than side of "+1", also known as from the left. For the given of "–a", "$\vee-\infty$" yields a revised power (c) which infinitely approaches "–1". If "–a < –1", this approach is from the lesser than side of "–1", also known as from the left. If "–a > –1", this approach is from the greater than side of "–1", also known as from the right. Unity elements of this type are "virtual unity elements".**

**Revised involution has the elimination element of "zero" for the givens of "a" or "b".**

**Revised involution has closure within the set of real numbers. However, the associative and commutative properties do not apply to revised involution.**

**The cancellation property applies to revised involution as a derivative property of involutive unity (i.e., the involution of the "virtual unity element" which in revised involution is either the positive infinitesimal or the negative infinitesimal). Incidentally, the indiscriminant cancellation of two unequal bases (a) on opposite sides of an equation is made unsound mathematically by precise relationships involving interactions with positive infinity and negative infinity under revised arithmetic.**

**Since the commutative property does not apply in revised involution, the cancellation property has two distinct expressions; one of which applies unconditionally and one of which applies conditionally but is convertible.**

**The distributive property for revised involution over revised multiplication applies if the two factors (a & b) involved are of the same sign, either originally or through conversion of identical multipliers (b). Potentially, this is in every case. The distributive property for revised involution over revised multiplication does not apply if the two factors (a & b) involved are of opposite signs until/unless they are converted to the same sign by providing the unknown, identical multiplier (b).**

**All of the applicable properties assembled necessitate the set of real numbers under revised involution to be a mathematical system. It is not a group.**

**The signs of the base (a) and the exponent (b) must be the same,**
**Either originally or through conversion of identical exponents (b),**
**for one set of missing variable formulae to be universal for "a", "b" & "c".**

**Where the base (a) has a domain of the complete set of real numbers and**
**the exponent (b) has a range of the complete set of real numbers,**
**the revised power (c) has a scope of ¼ of the set of real numbers-**
**positive <u>or</u> negative <u>&</u> having an absolute value greater than <u>or</u> lesser than "+1".**
**The sign of the base (a) likewise determines the sign of the revised power (c).**
**Whether the absolute value of the base (a) is greater than or lesser than "+1"**
**likewise determines whether the absolute value of the revised power (c)**
**is greater than or lesser than "+1".**

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**re: the exponential constant**

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**1.** $+e = +2.71828 \ldots$

$-e = -2.71828 \ldots$

**2.** $\vee +e = +0.36787 \ldots$

$\vee -e = -0.36787 \ldots$

**3.** $+e^{\vee +e} = +1.444667 \ldots$

$-e^{\vee -e} = -1.444667 \ldots$

**4.** $\vee +e^{\vee +e} = +0.6922 \ldots$

$\vee -e^{\vee -e} = -0.6922 \ldots$

**5.** $[+1 + \vee +\infty]^{+\infty} = +e$

$[-1 + \vee -\infty]^{-\infty} = -e$

**6.** $+e^{\vee +\infty} = +1 + \vee +\infty$

$-e^{\vee -\infty} = -1 + \vee -\infty$

**7.** $$[+1 + \vee -\infty]^{+\infty} = \vee +e$$

$$[-1 + \vee +\infty]^{-\infty} = \vee -e$$

**8.** $$\vee +e^{\vee +\infty} = +1 + \vee -\infty$$

$$\vee -e^{\vee -\infty} = -1 + \vee +\infty$$

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# revised analytic plane geometry

## part IV

**revised linear equations**

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**In revised algebra, there are four unique forms of revised linear, binomial equations (first degree). Of course, all linear equations, revised or conventional, must be solvable for either one of the two possible unknowns, "x" or "y". Thus, they exist in two forms just to isolate each possible unknown. Moreover, all revised linear equations have two unique formulae corresponding to the signs of the two knowns, "a" and "b", as the same or opposite.**

**Revised linear equations may be classified as a simplified, special case of linear functions wherein one of the two unknowns, "x" or "y", equals "zero" in every case and thus, is known or eliminated as an unknown. They always graph as a point on the rectangular coordinate system.**

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**revised linear equations (first degree)**

**master formulae**

**signs of knowns formulae**

---

**"xa" form**

**one unknown (x)**
**two knowns (a & b)**

$(\pm x)(\pm a) + b = 0$ **opposite signs (a & b)**

$(x)(a) + \wedge b = 0$ **same signs (a & b)**

---

**"(y)($\vee$b)" form**

**one unknown (y)**
**two knowns (a & b)**

$(\pm y)(\vee\pm b) + a = 0$ **opposite signs (a & b)**

$(y)(\vee b) + \wedge a = 0$ **same signs (a & b)**

---

**first degree equations**
**revised linear, binomial equations**

---

$(\pm x)(\pm a) + b = 0$ — **revised linear equation (first degree)**

**"xa" form**

**graph representation is as a point (x, y) on the x axis**

**one unknown (x)**
**two knowns of opposite signs (a & b)**

| | |
|---|---|
| given | a = a known real number. |
| given | b = a known real number. |
| | x = an unknown real number. |
| | y = zero (0). |
| | (x, y) = the point graph. |

---

(1) $(\pm x)(\pm a) + b = 0$ — **revised linear equation (first degree)**

(2) $(\pm x)(\pm a) = \wedge b$

(3) $b = \wedge[(\pm x)(\pm a)]$

(4) $x = \wedge b \;\; x \;\; \vee a$ — **revised linear formula**

**Given the values of "a" and "b" which are of opposite signs and knowing the value of "y" is set equal to "zero", the unknown value of "x", which is of the same sign as "a" and the opposite sign of "b", can be determined by using the revised linear formula in every case.**

**examples-**

| | | I | II |
|---|---|---|---|
| given | a | -2 | +4 |
| given | b | +6 | -8 |
| unknown | x | -3 | +2 |
| set | y | 0 | 0 |
| point | (x, y) | (-3, 0) | (+2, 0) |

**first degree equations**
**revised linear, binomial equations**

---

**$(\pm x)(\pm a) + \wedge b = 0$** — **revised linear equation (first degree)**

**"xa" form**

**graph representation is as a point (x, y) on the x axis**

**one unknown (x)**
**two knowns of the same sign (a & b)**

| | |
|---|---|
| given | a = a known real number. |
| given | b = a known real number. |
| | x = an unknown real number. |
| | y = zero (0). |
| | (x, y) = the point graph. |

---

**(1)** $(\pm x)(\pm a) + \wedge b = 0$ — **revised linear equation (first degree)**

**(2)** $(\pm x)(\pm a) = b$

**(3)** $x = b\ x \vee a$ — **revised linear formula**

**Given the values of "a" and "b" which are of the same sign and knowing the value of "y" is set equal to "zero", the unknown value of "x", which is of the same sign as "a" and "b", can be determined by using the revised linear formula in every case.**

**examples-**

| | | I | II |
|---|---|---|---|
| given | a | +2 | -3 |
| given | b | +8 | -6 |
| unknown | x | +4 | -2 |
| set | y | 0 | 0 |
| point | (x, y) | (+4, 0) | (-2, 0) |

**first degree equations**
**revised linear, binomial equations**

---

**$(\pm y)(\vee\pm b) + a = 0$** — **revised linear equation (first degree)**

**"(y)($\vee$b)" form**

**graph representation is as a point (x, y) on the y axis**

**one unknown (y)**
**two knowns of opposite signs (a & b)**

| | |
|---|---|
| given | a = a known real number. |
| given | b = a known real number. |
| | x = zero (0). |
| | y = an unknown real number. |
| | (x, y) = the point graph. |

---

**(1)** $(\pm y)(\vee\pm b) + a = 0$ — **revised linear equation (first degree)**

**(2)** $(\pm y)(\vee\pm b) = \wedge a$

**(3)** $a = \wedge[(\pm y)(\vee\pm b)]$

**(4)** $y = \wedge a \times b$ — **revised linear formula**

**Given the values of "a" and "b" which are of opposite signs and knowing the value of "x" is set equal to "zero", the unknown value of "y", which is of the same sign as "b" and the opposite sign of "a", can be determined by using the revised linear formula in every case.**

**examples-**

| | | I | II |
|---|---|---|---|
| given | a | +3 | -2 |
| given | b | -2 | +4 |
| set | x | 0 | 0 |
| unknown | y | -6 | +8 |
| point | (x, y) | (0, -6) | (0, +8) |

**first degree equations**
**revised linear, binomial equations**

---

**$(\pm y)(\vee\pm b) + \wedge a = 0$** — **revised linear equation (first degree)**

**"$(y)(\vee b)$" form**

**graph representation is as a point (x, y) on the y axis**

**one unknown (y)**
**two knowns of the same sign (a & b)**

| | |
|---|---|
| given | a = a known real number. |
| given | b = a known real number. |
| | x = zero (0). |
| | y = an unknown real number. |
| | (x, y) = the point graph. |

---

**(1)** $(\pm y)(\vee\pm b) + \wedge a = 0$ — **revised linear equation (first degree)**

**(2)** $(\pm y)(\vee\pm b) = a$

**(3)** $y = a \times b$ — **revised linear formula**

**Given the values of "a" and "b" which are of the same sign and knowing the value of "x" is set equal to "zero", the unknown value of "y", which is of the same sign as "a" and "b", can be determined by using the revised linear formula in every case.**

**examples-**

| | | I | II |
|---|---|---|---|
| given | a | -2 | +4 |
| given | b | -3 | +2 |
| set | x | 0 | 0 |
| unknown | y | -6 | +8 |
| point | (x, y) | (0, -6) | (0, +8) |

**revised linear equations (first degree)**

**rearrangements of master formulae**

---

**"xa" form**

**one unknown (x)**
**two knowns (a & b)**

**opposite signs (a & b)**

$$(\pm x)(\pm a) + b = 0$$

$$(\pm a)(\pm x) + b = 0$$

---

$$(\pm x)(\pm a) = \wedge b$$

$$(\pm a)(\pm x) = \wedge b$$

---

$$x = \wedge b \;\; x \;\; \vee a$$

---

$$a = \wedge b \;\; x \;\; \vee x$$

---

$$b = \wedge[(\pm x)(\pm a)]$$

$$b = \wedge[(\pm a)(\pm x)]$$

**xa" form**

**one unknown (x)**
**two knowns (a & b)**

**same signs (a & b)**

$$(x)(a) + \wedge b = 0$$

$$(a)(x) + \wedge b = 0$$

---

$$(x)(a) = b$$

$$(a)(x) = b$$

---

$$x = b \quad x \vee a$$

---

$$a = b \quad x \vee x$$

---

**"(y)(∨b)" form**

**one unknown (y)**
**two knowns (a & b)**

**opposite signs (a & b)**

$$(\pm y)(\vee\pm b) + a = 0$$

$$(\vee\pm b)(\pm y) + a = 0$$

---

$$(\pm y)(\vee\pm b) = \wedge a$$

$$(\vee\pm b)(\pm y) = \wedge a$$

---

$$y = \wedge a \times b$$

---

$$b = \wedge\vee a \times y$$

---

$$a = \wedge[(\pm y)(\vee\pm b)]$$

$$a = \wedge[(\vee\pm b)(\pm y)]$$

**"(y)(∨b)" form**

**one unknown (y)**
**two knowns (a & b)**

**same signs (a & b)**

$$(y)(\vee b) + \wedge a = 0$$

$$(\vee b)(y) + \wedge a = 0$$

---

$$(y)(\vee b) = a$$

$$(\vee b)(y) = a$$

---

$$y = a \times b$$

---

$$b = \vee a \times y$$

---

**revised "second degree" equations**

---

**In conventional algebra, the binomial theorem dictates that polynomial equations of successive degrees result directly from the repeated conventional multiplication of a binomial by itself an appropriate number of times as indicated by the exponent (b) in conventional involution.**

**To maintain comparable capabilities in revised algebra, the repeated revised multiplication of a binomial by itself an appropriate number of times as indicated by the exponent (b) in revised involution would seem a logical approach. Perhaps this would construct a series of equations of various degrees in revised algebra isomorphic and analog to the polynomial equations, represented by binomial equations of various degrees in conventional algebra.**

**Ultimately, any revised "second degree" equation constructed via the revised cross-multiplication of the appropriate revised first degree equation is reducible to and solvable as its original, first degree equation under revised algebra.**

**This holds true in all four forms of revised linear, binomial equations, solved for the unknown of either "x" or "y" and where the knowns "a" and "b" are either of the same sign or opposite signs. A proof using the expansion of a simplified, universal formula of a revised algebra equation to the second degree follows. It exists as the foundation for an inductive proof whereby revised linear equations to the nth degree (any arbitrary degree) can be proven reducible to and equivalent with revised linear equations of the first degree in every case. Hence, in revised algebra, it is inaccurate hereafter to even refer to revised linear equations as being of various degrees and redundant to describe them as being of the first degree or binomial.**

**Comparatively, in conventional algebra, solutions to polynomial equations of the fifth degree and higher are generally impossible to determine although they still exist theoretically. This shortcoming is a strong argument against the maximum applicability which is routinely attributed to conventional mathematics as a whole. Incidentally, although polynomial equations exist in revised algebra, they are not of special importance since they are not a consequence of or related to revised linear equations.**

**Although the demonstrated construction of a revised "second degree" equation is a complete failure, the foregoing material is left in place nonetheless as a valid proof of the vast and superior capabilities of a symmetrical, revised linear equation which cannot and need not be expanded to higher degrees.**

**revised "second degree" equation**
**simplified, universal formula**

---

**a = the first unknown addend**
**b = the second unknown addend**

---

**(1)** $a + b = 0$ **revised first degree equation**

**(2)** $(a + b)(a + b) = (0)(0)$ **revised "second degree" equation**

**(3)** $aa + ab + ba + bb = 0$

---

**subproof-** $ab + ba = 0$

**(a)** $a = \wedge b$

**revised first degree equation (rearrangement)**

**(b)** $ab + ba = (\wedge b)(b) + (b)(\wedge b)$

**substitution property**

**(c)** $(\wedge b)(b) + \wedge[(\wedge b)(b)] = 0$

**elimination element of addition**

**(d)** $\wedge[(\wedge b)(b)] = (b)(\wedge b)$

**distributive property for opposition over revised multiplication**

**(e)** $(\wedge b)(b) + (b)(\wedge b) = 0$

**substitution property**

**(f)** $ab + ba = 0$

**transitive property**

---

**(4)** $aa + bb = 0$

**(5)** $a + b = 0$ **revised first degree equation**

**revised linear functions**

---

**In revised analytic plane geometry, there are four unique forms of revised linear functions, analogous in basic structure to the revised linear equations. All linear functions, revised or conventional, relate the two unknowns, "x" and "y", in an infinite set of one-to-one correspondences. They exist in two forms just to isolate each unknown. Moreover, all revised linear functions in revised analytic plane geometry have two unique formulae corresponding to pairs of opposite quadrants, "I & III" or "II & IV", which a straight line must lie in.**

**Revised linear functions are an algebraic generalization of revised linear equations having two unknowns instead of one unknown. They always graph as a straight line on the rectangular coordinate system.**

---

**revised linear functions**
**master formulae**
**opposite quadrants formulae**

---

**"xa" form**

**two unknowns (x & y)**
**two knowns (a & b)**

| | |
|---|---|
| $(\pm x)(\pm a) + b = y$ | **quadrants I & III** |
| $(\mp x)(\mp a) + \wedge b = \wedge y$ | **quadrants II & IV** |

---

**"(y)(∨b)" form**

**two unknowns (x & y)**
**two knowns (a & b)**

| | |
|---|---|
| $(\pm y)(\vee\pm b) + a = x$ | **quadrants I & III** |
| $(\mp y)(\vee\mp b) + \wedge a = \wedge x$ | **quadrants II & IV** |

---

**revised linear functions**

**rearrangements of master formulae**

---

**"xa" form**

**two unknowns (x & y)**
**two knowns (a & b)**

**quadrants I & III**

$$(\pm x)(\pm a) + b = y$$

$$(\pm a)(\pm x) + b = y$$

---

$$x = (y + \wedge b)\ x\ \vee a$$

---

$$a = (y + \wedge b)\ x\ \vee x$$

---

$$b = \wedge[(\pm x)(\pm a)] + y$$

$$b = \wedge[(\pm a)(\pm x)] + y$$

---

$$(\pm x)(\pm a) + b + \wedge y = 0$$

$$(\pm a)(\pm x) + b + \wedge y = 0$$

---

**"xa" form**

**two unknowns (x & y)**
**two knowns (a & b)**

**quadrants II & IV**

$$(\mp x)(\mp a) + \wedge b = \wedge y$$

$$(\mp a)(\mp x) + \wedge b = \wedge y$$

---

$$x = (\wedge y + b)\ x\ \vee a$$

---

$$a = (\wedge y + b)\ x\ \vee x$$

---

$$b = (\mp x)(\mp a) + y$$

$$b = (\mp a)(\mp x) + y$$

---

$$y = \wedge[(\mp x)(\mp a)] + b$$

$$y = \wedge[(\mp a)(\mp x)] + b$$

---

$$(\mp x)(\mp a) + \wedge b + y = 0$$

$$(\mp a)(\mp x) + \wedge b + y = 0$$

---

**"(y)(∨b)" form**

**two unknowns (x & y)**
**two knowns (a & b)**

**quadrants I & III**

$$(\pm y)(\vee\pm b) + a = x$$

$$(\vee\pm b)(\pm y) + a = x$$

---

$$y = (x + \wedge a) \times b$$

---

$$b = \vee[(x + \wedge a)(\vee y)]$$

---

$$a = \wedge[(\pm y)(\vee\pm b)] + x$$

$$a = \wedge[(\vee\pm b)(\pm y)] + x$$

---

$$(\pm y)(\vee\pm b) + a + \wedge x = 0$$

$$(\vee\pm b)(\pm y) + a + \wedge x = 0$$

---

**"(y)(∨b)" form**

**two unknowns (x & y)**
**two knowns (a & b)**

**quadrants II & IV**

$$(\mp y)(\vee\mp b) + \wedge a = \wedge x$$

$$(\vee\mp b)(\mp y) + \wedge a = \wedge x$$

---

$$y = (\wedge x + a)\ x\ b$$

---

$$b = \vee[(\wedge x + a)(\vee y)]$$

---

$$a = (\mp y)(\vee\mp b) + x$$

$$a = (\vee\mp b)(\mp y) + x$$

---

$$x = \wedge[(\mp y)(\vee\mp b)] + a$$

$$x = \wedge[(\vee\mp b)(\mp y)] + a$$

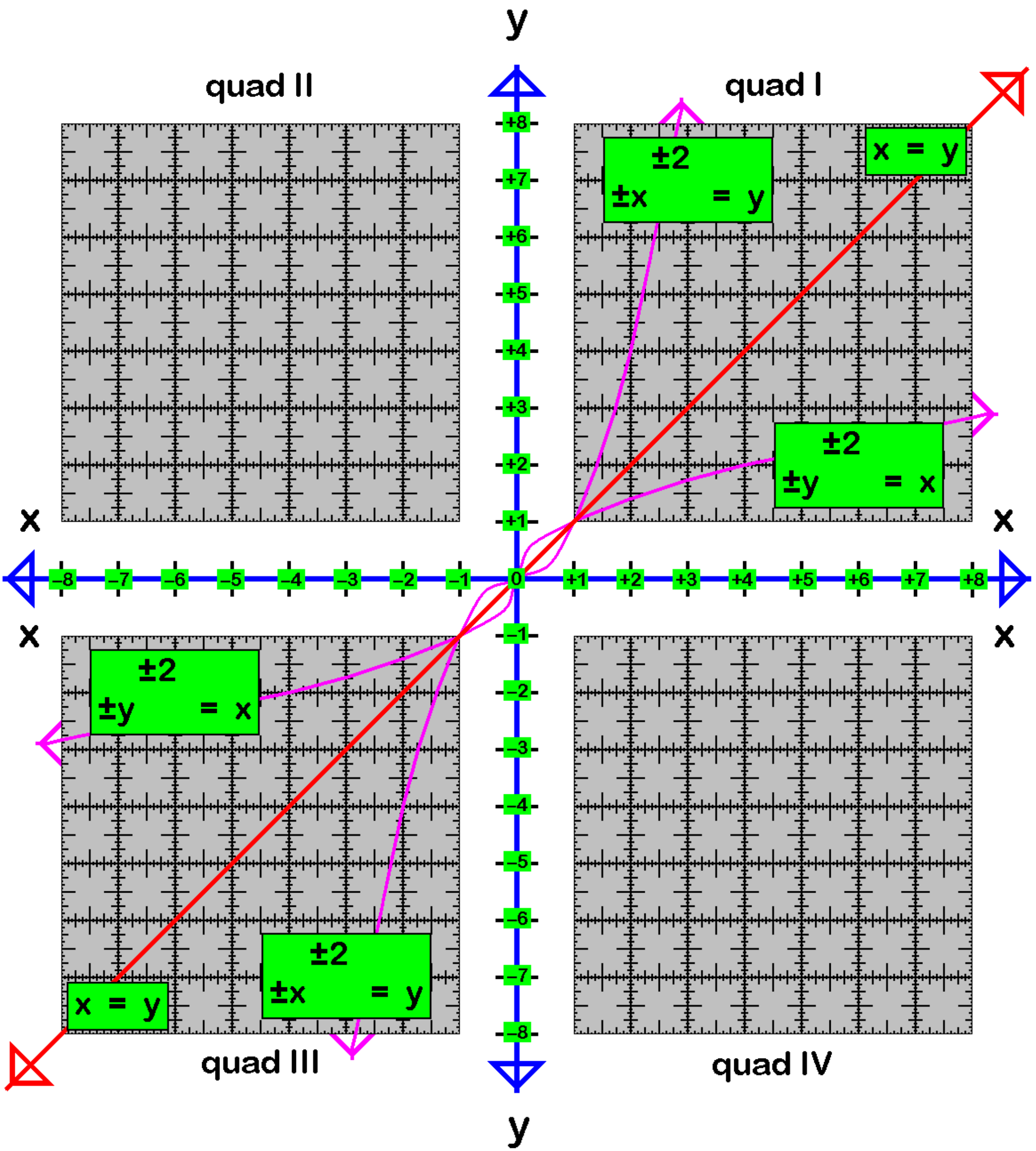
---

$$(\mp y)(\vee\mp b) + \wedge a + x = 0$$

$$(\vee\mp b)(\mp y) + \wedge a + x = 0$$

---

# revised power functions



**revised power functions**
**representative points for examples within graph**

| | |
|---|---|
| $\pm y^{\pm 2} = x$<br>$\pm b^{\pm 2} = a$ | $\pm x^{\pm 2} = y$<br>$\pm a^{\pm 2} = b$ |
| (0, 0) | (0, 0) |
| $(\pm 0.\overline{3}, \pm 0.\overline{1})$ | $(\pm 0.\overline{1}, \pm 0.\overline{3})$ |
| (±0.5, ±0.25) | (±0.25, ±0.5) |
| (±0.71, ±0.5) | (±0.5, ±0.71) |
| (±1, ±1) | (±1, ±1) |
| (±1.41, ±2) | (±2, ±1.41) |
| (±1.73, ±3) | (±3, ±1.73) |
| (±2, ±4) | (±4, ±2) |
| (±3, ±9) | (±9, ±3) |

**revised power functions**

---

**a = x = base #1 which may be with or without the exponent (n), the real number variables from the x axis.**

**b = y = base #2 which may be with or without the exponent (n), the real number variables from the y axis.**

**n = the exponent to either base #1 (a) <u>or</u> base #2 (b), a positive-over-negative or negative-over-positive real number constant.**

---

**two general forms of revised power functions**

$$a^{n} = b \quad \text{(exponent to base "a" form)}$$

$$b^{n} = a \quad \text{(exponent to base "b" form)}$$

---

**Since "a" and "b" are both bases, they must always be of the same signs as one another, even when the sign of one of them reverses, for equations to be balanced. Hence, all revised power functions graph only in quadrants I & III where "a" and "b" are of the same sign on the coordinate plane.**

**All revised power functions are continuous.**

**For each of the two general forms of revised power functions, applying to the respective bases (a & b), there are two possible relations between the original signs of a base (a or b) and the exponent (n)- either the same or opposite. If opposite signs exist originally, conversion of identical exponents is <u>always</u> recommended to obtain the same signs and be able to use the simpler formulae.**

**Generalized into correspondent notation, there are four combinations, two pairs of which are equivalent. Hence, any single, revised power function can be expressed, in simplified terms, by four equivalent equations, none of which are any more or less proper than the others.**

---

**4 equivalent equations of a revised power function**

| | column A | column B |
|---|---|---|
| | same signs<br>form<br>$\pm a^{\pm n}$ | same signs<br>form<br>$\pm b^{\pm n}$ |
| block I | ---------- | ---------- |
| 1.<br>2. | $\pm b = \pm a^{\pm n}$<br>$\mp b = \mp a^{\mp n}$ | $\pm a = \pm b^{\pm n}$<br>$\mp a = \mp b^{\mp n}$ |
| block II | ---------- | ---------- |
| 1.<br>2. | $\pm a = \pm b^{\vee \pm n}$<br>$\mp a = \mp b^{\vee \mp n}$ | $\pm b = \pm a^{\vee \pm n}$<br>$\mp b = \mp a^{\vee \mp n}$ |

**examples-**

**four equivalent equations**

$$\pm y = \pm x^{\pm b} \qquad \text{column A}$$

$$\pm b = \pm 2 \qquad \text{given}$$

---

$$\pm y = \pm x^{\pm 2}$$

$$\mp y = \mp x^{\mp 2}$$

---

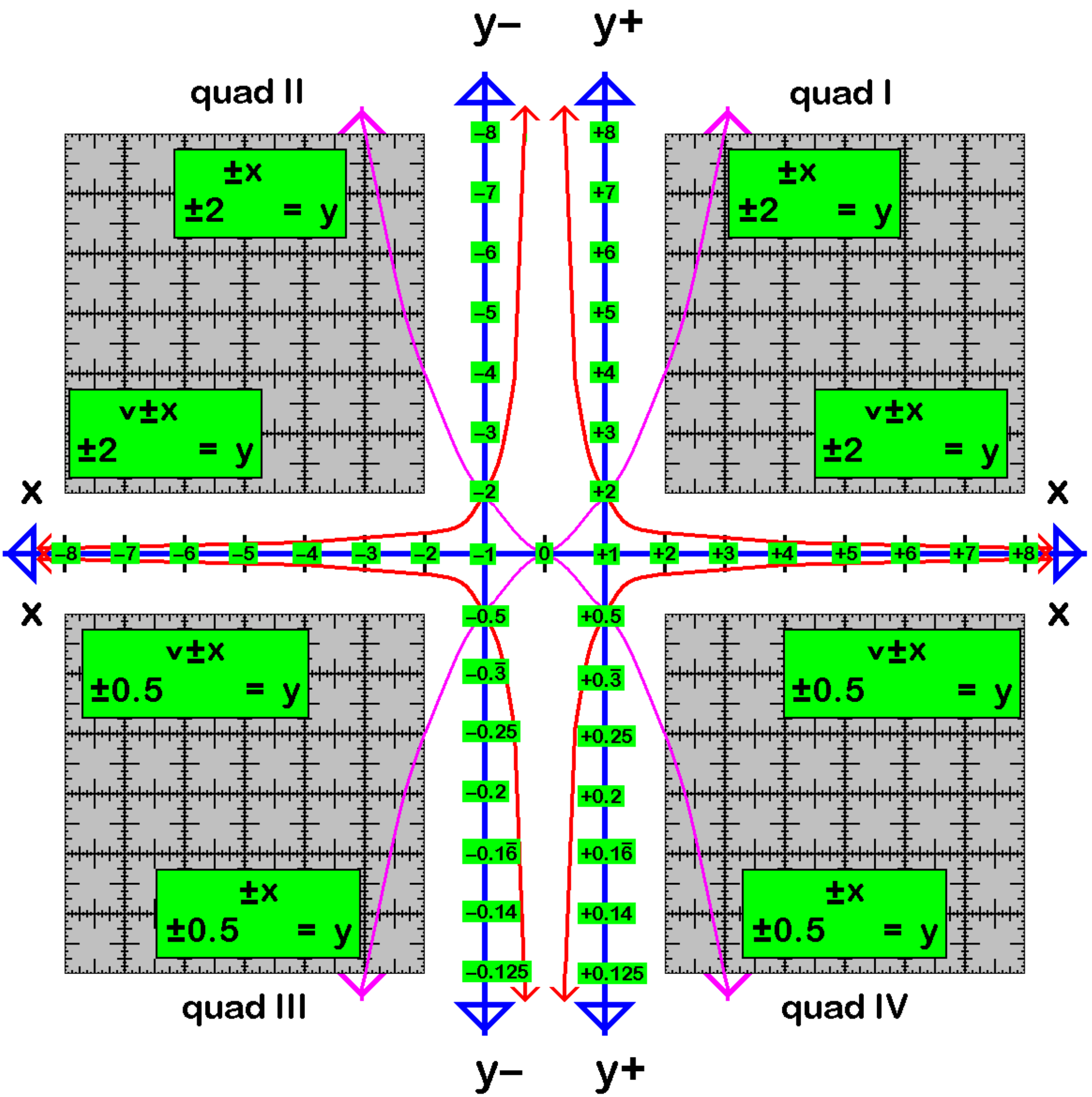
$$\pm x = \pm y^{\pm 0.5}$$

$$\mp x = \mp y^{\mp 0.5}$$

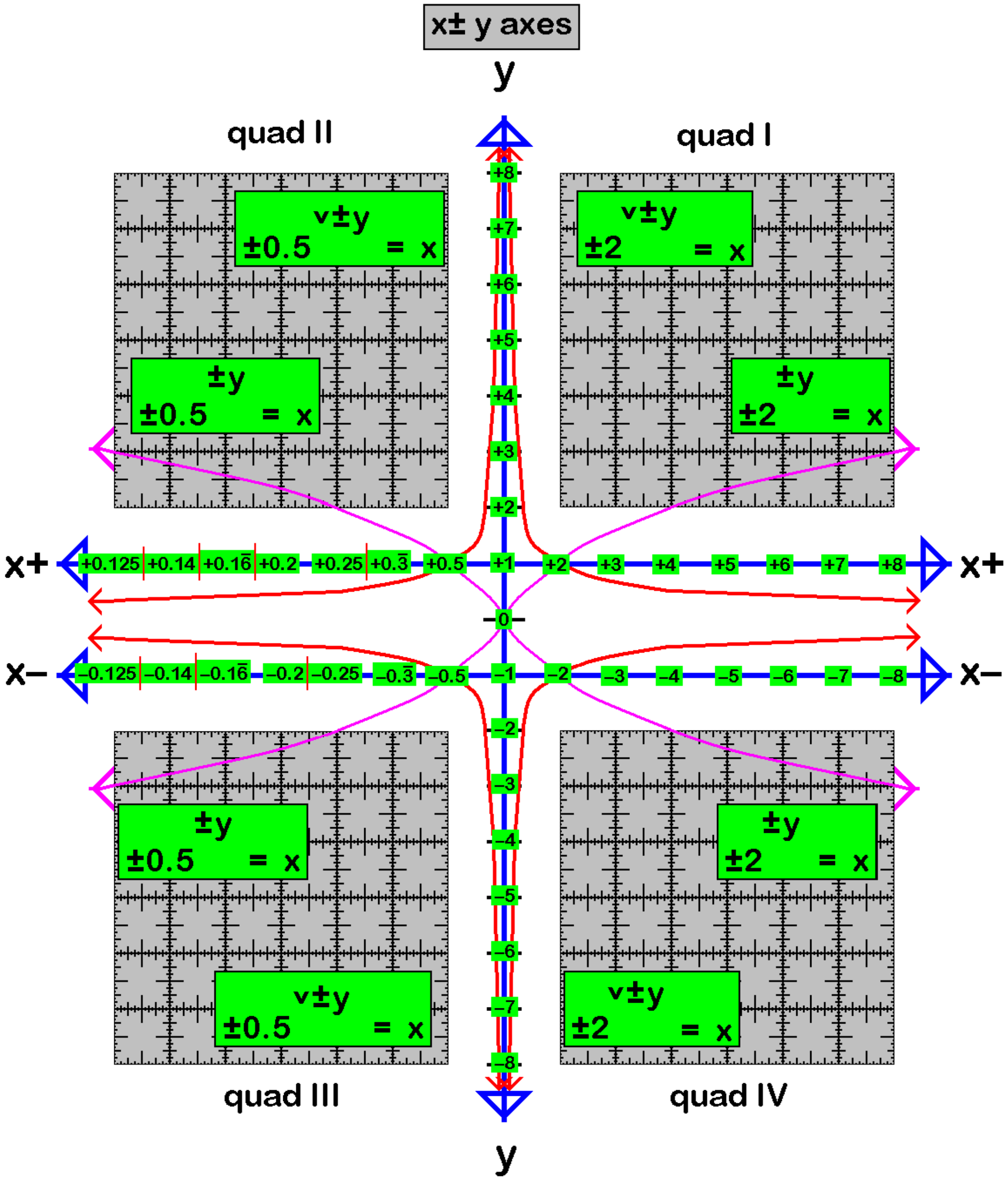
---

# revised exponential-logarithmic functions

## x y± axes



# revised exponential-logarithmic functions



**revised exponential/logarithmic functions**

**representative points for examples within two graphs**

**x y± axes graph**

**x± y axes graph**

---

**x y± axes graph**
**exponent "x" form**

---

$$\pm 2^{\pm x} = y$$

$$\pm 2^{\pm a} = b$$ **a continuous function**

---

$$a = \log \pm b \times \vee \log \pm 2$$

---

**quadrants I & II**

---

**(0, 0)**

**(+1, +2)**

**(+2, +4)**

**(+3, +8)**

**(–3, –8)**

**(–2, –4)**

**(–1, –2)**

---

---

**x y± axes graph**
**exponent "x" form**

---

$$\pm 0.5^{\pm x} = y$$

$$\pm 0.5^{\pm a} = b$$ **a continuous function**

---

$$a = \log \pm b \ x \ \vee \log \pm 0.5$$

---

**quadrants III & IV**

---

**(0, 0)**

**(–1, –0.5)**

**(–2, –0.25)**

**(–3, –0.125)**

**(+3, +0.125)**

**(+2, +0.25)**

**(+1, +0.5)**

---

---

**x y± axes graph**
**exponent "x" form**

---

$$\pm 2^{\vee \pm x} = y$$

$$\pm 2^{\vee \pm a} = b$$ **a discontinuous function**

---

$$a = \vee \log \pm b \times \log \pm 2$$

---

| quadrant I | quadrant II |
|---|---|
| (+8, +1.09) | (–8, –1.09) |
| (+4, +1.19) | (–4, –1.19) |
| (+2, +1.41) | (–2, –1.41) |
| (+1, +2) | (–1, –2) |
| (+0.5, +4) | (–0.5, –4) |
| $(+0.\overline{3}, +8)$ | $(-0.\overline{3}, -8)$ |
| (0, 0) | (0, 0) |

---

**x y± axes graph**
**exponent "x" form**

---

$$\pm 0.5^{\vee \pm x} = y$$

$$\pm 0.5^{\vee \pm a} = b$$ **a discontinuous function**

---

$$a = \vee \log \pm b \times \log \pm 0.5$$

---

| quadrant III | quadrant IV |
|---|---|
| (–8, –0.92) | (+8, +0.92) |
| (–4, –0.84) | (+4, +0.84) |
| (–2, –0.71) | (+2, +0.71) |
| (–1, –0.5) | (+1, +0.5) |
| (–0.5, –0.25) | (+0.5, +0.25) |
| $(-0.\overline{3}, -0.125)$ | $(+0.\overline{3}, +0.125)$ |
| (0, 0) | (0, 0) |

---

**x± y axes graph**
**exponent "y" form**

---

$$\pm 2^{\pm y} = x$$

$$\pm 2^{\pm b} = a$$ **a continuous function**

---

$$b = \log \pm a \; x \vee \log \pm 2$$

---

**quadrants I & IV**

---

**(0, 0)**

**(+2, +1)**

**(+4, +2)**

**(+8, +3)**

**(–8, –3)**

**(–4, –2)**

**(–2, –1)**

---

---

**x± y axes graph**
**exponent "y" form**

---

$$\pm 0.5^{\pm y} = x$$

$$\pm 0.5^{\pm b} = a$$ **a continuous function**

---

$$b = \log \pm a \ x \ \vee \log \pm 0.5$$

---

**quadrants II & III**

---

**(0, 0)**

**(+0.125, +3)**

**(+0.25, +2)**

**(+0.5, +1)**

**(–0.5, –1)**

**(–0.25, –2)**

**(–0.125, –3)**

---

---

**x± y axes graph**
**exponent "y" form**

---

$$\pm 2^{\vee \pm y} = x$$

$$\pm 2^{\vee \pm b} = a$$ **a discontinuous function**

---

$$b = \vee \log \pm a \times \log \pm 2$$

---

| quadrant I | quadrant IV |
|---|---|
| (+1.09, +8) | (–1.09, –8) |
| (+1.19, +4) | (–1.19, –4) |
| (+1.41, +2) | (–1.41, –2) |
| (+2, +1) | (–2, –1) |
| (+4, +0.5) | (–4, –0.5) |
| (+8, $+0.\overline{3}$) | (–8, $-0.\overline{3}$) |
| (0, 0) | (0, 0) |

---

**x± y axes graph**
**exponent "y" form**

---

$$\pm 0.5^{\vee \pm y} = x$$

$$\pm 0.5^{\vee \pm b} = a$$ **a discontinuous function**

---

$$b = \vee \log \pm a \times \log \pm 0.5$$

---

| quadrant II | quadrant III |
|---|---|
| (+0.92, +8) | (–0.92, –8) |
| (+0.84, +4) | (–0.84, –4) |
| (+0.71, +2) | (–0.71, –2) |
| (+0.5, +1) | (–0.5, –1) |
| (+0.25, +0.5) | (–0.25, –0.5) |
| (+0.125, +0.$\overline{3}$) | (–0.125, –0.$\overline{3}$) |
| (0, 0) | (0, 0) |

**revised exponential/logarithmic functions**

---

**a = x = The general form of the function determines whether it is as a base without an exponent <u>or</u> an exponent to the base "n". It is a real number variable from the x axis or the x± axes.**

**b = y = The general form of the function determines whether it is as a base without an exponent <u>or</u> an exponent to the base "n". It is a real number variable from the y axis or the y± axes.**

**n = The base to the exponent, either "a" or "b", whichever is not serving as a base without an exponent. It is a positive <u>and</u> negative real number constant which defines the function.**

---

**two general forms of revised exponential/logarithmic functions**

$$n^{x} = y \qquad \text{(exponent "x" form)}$$

$$n^{y} = x \qquad \text{(exponent "y" form)}$$

---

**The appropriate general form of exponential/logarithmic functions is most important since it immediately determines which of the two graphs with their contrasting, numerical axes schemes is readily usable.**
**In the "exponent 'x' form", there is the x axis and two y axes, the y+ axis and y– axis. In the "exponent 'y' form", there are two x axes, the x+ axis and x– axis, and the y axis.**

**Half of all revised exponential/logarithmic functions are continuous and half are discontinuous. This is true for whichever of the two graphs is in use.**

**For each of the two general forms of revised exponential/logarithmic functions, determined by the exponent (a or b), there are two possible relations between the signs of base (n) and the exponent (a or b)- either as the same or opposite. If opposite signs exist originally, conversion of identical exponents is <u>always</u> recommended to obtain the same signs and be able to use the simpler formulae.**

**Generalized into correspondent notation, there are four combinations, two pairs of which are equivalent. Hence, any single, revised exponential/logarithmic function can be expressed, in simplified terms, by four equivalent equations, none of which are more or less proper than the others.**

_____

**4 equivalent equations of a revised exponential/logarithmic function**

| | column A | column B |
|---|---|---|
| | same signs<br>form<br>$\pm n^{\pm a}$ | same signs<br>form<br>$\pm n^{\pm b}$ |
| block I | ---------- | ---------- |
| 1.<br>2. | $\pm b = \pm n^{\pm a}$<br>$\mp b = \mp n^{\mp a}$ | $\pm a = \pm n^{\pm b}$<br>$\mp a = \mp n^{\mp b}$ |
| block II | ---------- | ---------- |
| 1.<br>2. | $\pm a = \log \pm b \times \vee\log \pm n$<br>$\mp a = \log \mp b \times \vee\log \mp n$ | $\pm b = \log \pm a \times \vee\log \pm n$<br>$\mp b = \log \mp a \times \vee\log \mp n$ |

**examples-**

**four equivalent equations**

$$\pm y = \pm n^{\pm x} \qquad \textbf{column A}$$

$$\pm n = \pm 4 \qquad \textbf{given}$$

---

$$\pm y = \pm 4^{\pm x}$$

$$\mp y = \mp 4^{\mp x}$$

---

$$\pm x = \log \pm y \;\; x \;\; \vee \log \pm 4$$

$$\mp x = \log \mp y \;\; x \;\; \vee \log \mp 4$$

---

**appendix I-**

## infinity equations

---

**The predictive capability, accuracy and numerical consistency of the extended real number continuum model for every real number under the given unary operations can be applied without limitation to the infinite values (i.e., positive infinity, positive infinitesimal, negative infinity, negative infinitesimal) as well. By extension, the treatment of the infinite values under the binary operations of revised arithmetic exactly as every other positive and negative real number as well as zero is a responsible method that supplements and is consistent with accepted methods (already developed by other mathematicians) for handling surreal numbers.**

**In any event, a set-theoretical basis for the infinite values is equally speculative and harder to justify than an arithmetic basis since a set-theoretical basis allows the infinite values to break rules of arithmetic which every other real number must abide by.**

**There are three types of infinity equations covered in this work: unary, binary and numerical.**

**The 12 unary infinity equations are the select, special cases that involve infinite values ($+\infty$, $-\infty$, $\vee+\infty$, $\vee-\infty$). The unary operations of extended real numbers are opposition and/or reciprocation. They are taken directly from the graph "the extended real number continuum" in a likewise manner as any other real number.**

**It is important to note, however, that the placement of the infinite values is not true to scale. In fact, they are graphically represented vastly farther from zero than is accurate. This is necessary since they <u>exist</u> infinitely close to zero at exactly one point's distance. Obviously, the accurate depiction would not be visible or distinguishable.**

**The 8 binary infinity equations treated are direct interactions of two, unique infinite values ($+\infty$, $-\infty$, $\vee+\infty$, $\vee-\infty$) under only the revised binary operations of addition and revised multiplication. All are rearrangements of a given unary infinity equation.**

**The 36 numerical infinity equations are based upon laws of revised arithmetic that hold true for every other extended real number. They involve either identity or elimination elements within addition, revised multiplication or revised involution.**

**A grand total of 56 reducible infinity equations are given within this work, many of which are of indeterminate form in conventional mathematics.**

---

**I. unary infinity equations**

| | |
|---|---|
| opposition | 4 |
| reciprocation | 4 |
| opposition <u>and</u> reciprocation | 4 |
| subtotal- | 12 |

**II. binary infinity equations**

| | |
|---|---|
| addition | 4 |
| revised multiplication | 4 |
| subtotal- | 8 |

**III. numerical infinity equations**

| | |
|---|---|
| addition | |
| identity element (as "a" & "b") | 8 |
| revised multiplication | |
| identity element (as "a" & "b") | 8 |
| elimination element (as "a" & "b") | 8 |
| revised involution | |
| identity element (as "b" only) | 4 |
| elimination element (as "a" & "b") | 8 |
| subtotal- | 36 |
| grand total- | 56 |

**the extended real number continuum**

**unary operations**
**(opposition and/or reciprocation)**

**unary infinity equations**
**(and other special cases)**

---

**opposition** **I**

| | |
|---|---|
| $\wedge -1 = +1$ | A |
| $\wedge +1 = -1$ | B |
| $\wedge -\infty = +\infty$ | C |
| $\wedge +\infty = -\infty$ | D |
| $\wedge 0 = 0$ | E |
| $\wedge(\vee -\infty) = \vee +\infty$ | F |
| $\wedge(\vee +\infty) = \vee -\infty$ | G |

---

**reciprocation** **II**

| | |
|---|---|
| $\vee -1 = -1$ | A |
| $\vee(-\infty) = \vee -\infty$ | B |
| $\vee(\vee -\infty) = -\infty$ | C |
| $\vee 0 = 0$ | D |
| $\vee(+\infty) = \vee +\infty$ | E |
| $\vee(\vee +\infty) = +\infty$ | F |
| $\vee +1 = +1$ | G |

---

**opposition <u>and</u> reciprocation** **III**

---

| | |
|---|---|
| $\wedge\vee -1 = +1$ | **A** |
| $\wedge\vee -\infty = \vee +\infty$ | **B** |
| $\wedge\vee(\vee +\infty) = -\infty$ | **C** |
| $\wedge\vee 0 = 0$ | **D** |
| $\wedge\vee +\infty = \vee -\infty$ | **E** |
| $\wedge\vee(\vee -\infty) = +\infty$ | **F** |
| $\wedge\vee +1 = -1$ | **G** |

---

**binary infinity equations**

**(related forms of unary infinity equations)**

---

**addition**

| | |
|---|---|
| $n + \wedge n = 0$ | **general law** |
| $+\infty + -\infty = 0$ | **(re: unary I-C)** |
| $-\infty + +\infty = 0$ | **(re: unary I-D)** |
| $\vee +\infty + \vee -\infty = 0$ | **(re: unary I-F)** |
| $\vee -\infty + \vee +\infty = 0$ | **(re: unary I-G)** |

**revised multiplication**

| | |
|---|---|
| $\pm n \times \vee \pm n = \pm 1$ | **general law** |
| $\vee -\infty \times -\infty = -1$ | **(re: unary II-B)** |
| $-\infty \times \vee -\infty = -1$ | **(re: unary II-C)** |
| $\vee +\infty \times +\infty = +1$ | **(re: unary II-E)** |
| $+\infty \times \vee +\infty = +1$ | **(re: unary II-F)** |

**numerical infinity equations**

**addition**
**- identity element**

---

**$n + 0 = n$** **general laws**

**$0 + n = n$**

---

**$+\infty + 0 = +\infty$**

**$\vee+\infty + 0 = \vee+\infty$**

**$\vee-\infty + 0 = \vee-\infty$**

**$-\infty + 0 = -\infty$**

---

**revised multiplication**
**- identity element**

---

**$n \times \pm 1 = n$** **general law**

---

**$+\infty \times \pm 1 = +\infty$**

**$\vee+\infty \times \pm 1 = \vee+\infty$**

**$\vee-\infty \times \pm 1 = \vee-\infty$**

**$-\infty \times \pm 1 = -\infty$**

---

$\pm 1 \times \pm n = \pm n$ **general law**

$+1 \times +n = +n$ **-positive application**

---

$+1 \times +\infty = +\infty$

$+1 \times \vee +\infty = \vee +\infty$

---

$\pm 1 \times \pm n = \pm n$ **general law**

$-1 \times -n = -n$ **-negative application**

---

$-1 \times -\infty = -\infty$

$-1 \times \vee -\infty = \vee -\infty$

---

**revised multiplication**
**- elimination element**

---

$n \times 0 = 0$ **general laws**

$0 \times n = 0$

---

$+\infty \times 0 = 0$

$\vee +\infty \times 0 = 0$

$\vee -\infty \times 0 = 0$

$-\infty \times 0 = 0$

---

$0 \times +\infty = 0$

$0 \times \vee +\infty = 0$

$0 \times \vee -\infty = 0$

$0 \times -\infty = 0$

---

**revised involution**
**- identity element**

---

$$n^{\pm 1} = n$$ **general law**

---

$$+\infty^{\pm 1} = +\infty$$

$$\vee+\infty^{\pm 1} = \vee+\infty$$

$$\vee-\infty^{\pm 1} = \vee-\infty$$

$$-\infty^{\pm 1} = -\infty$$

---

**revised involution**
**- elimination element**

---

$$n^{0} = 0$$ **general law**

---

$$+\infty^{0} = 0$$

$$\vee+\infty^{0} = 0$$

$$\vee-\infty^{0} = 0$$

$$-\infty^{0} = 0$$

---

$$0^{n} = 0$$ **general law**

---

$$0^{+\infty} = 0$$

$$0^{\vee+\infty} = 0$$

$$0^{\vee-\infty} = 0$$

$$0^{-\infty} = 0$$

---

**appendix II-**

**accommodating extremely large numbers**
**with higher, revised binary operations**

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***n = any revised binary operation**

---

**+ <u>or</u> *1 = addition- the first binary operation (conventional).**

**a + b = c**

**OR**

**a (*1) b = c**

---

**x <u>or</u> *2 = revised multiplication- the second revised binary operation.**

**a x b = c**

**OR**

**a (*2) b = c**

---

***3 = revised involution- the third revised binary operation.**

$$a^{b} = c$$

**OR**

**a (*3) b = c**

---

***4 = revised hyper-involution- the fourth revised binary operation.**

**a (*4) b = c**

---

**Revised hyper-involution is based upon repeated, revised involution in an analogous manner as revised involution is based upon repeated, revised multiplication and so forth.**

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**revised hyper-involution (*4)**

---

**examples-**

**–3 (*4) 0 = 0**

**–3 (*4) –1 = –3**

**–3 (*4) –2 = –3 (*3) –3 = –27**

**–3 (*4) –3 = –3 (*3) –3 (*3) –3 = –19683**

---

**–3 (*4) –4 = –3 (*3) –3 (*3) –3 (*3) –3**

$$= -7.62 \times -10^{-12}$$

---

---

**revised hyper-involution (*4) equated to scientific notation**

---

**examples-**

$$+10 \ (*4) \ 0 = 0$$

$$+10 \ (*4) \ \pm 1 = +10$$

$$+10 \ (*4) \ +2 = +10^{+10} = +10{,}000{,}000{,}000$$

$$+10 \ (*4) \ +3 = (+10^{+10^{+10}}) = +10^{+100}$$

$$+10 \ (*4) \ +4 = [\,(+10^{+10^{+10^{+10}}})\,] = +10^{+1000}$$

---

**The last example value is greater than Skewes' number.**

**There is really no point in going further unless one is intent upon pursuing astronomical, combinatoric values- the highest known of which is Graham's number. For such an impractical mission, I recommend much higher, revised binary operations of which there are a theoretically-unlimited number which can be built in a perfectly likewise manner as those demonstrated.**

**As a number crunching principle, raising the revised binary operation is much more efficient than raising the second variable (b) which, in turn, is more efficient than raising the first variable (a). Of course, this is assuming that neither variable (a or b) is prohibitively close to "±1" .**

---

---

**revised *5 equated to scientific notation**

---

**examples-**

**+10 (*5) 0 = 0**

**+10 (*5) ±1 = +10**

---

**+10 (*5) +2 = +10 (*4) +10**

$$= +10^{+1,000,000,000}$$

---

---

**revised *5 - *10 equated to next lower revised binary operation**

---

**examples-**

**+10 (*5) +2 = +10 (*4) +10**

**+10 (*6) +2 = +10 (*5) +10**

**+10 (*7) +2 = +10 (*6) +10**

**+10 (*8) +2 = +10 (*7) +10**

**+10 (*9) +2 = +10 (*8) +10**

**+10 (*10) +2 = +10 (*9) +10**

---

---

**revised *10 - *1,000,000**
**lightly enriched for number crunching**

---

**examples-**

**+10 (*10) +1**

**+10 (*10) +100**

**+10 (*10) +10,000**

---

**+100 (*10) +10,000**

**+100 (*100) +10,000**

**+100 (*1000) +10,000**

**+100 (*10,000) +10,000**

**+100 (*100,000) +10,000**

**+100 (*1,000,000) +10,000**

---

---

**revised *1,000,000 - *1,000,000,000,000,000,000,000,000**
**heavily enriched for number crunching**

---

**examples-**

**+100 (*1,000,000) +10,000**

**+100 (*1,000,000) +1,000,000**

---

**+1000 (*1,000,000) +1,000,000**

**+1000 (*1,000,000,000) +1,000,000**

**+1000 (*1,000,000,000,000) +1,000,000**

**+1000 (*1,000,000,000,000,000) +1,000,000**

**+1000 (*1,000,000,000,000,000,000) +1,000,000**

**+1000 (*1,000,000,000,000,000,000,000) +1,000,000**

**+1000 (*1,000,000,000,000,000,000,000,000) +1,000,000**

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**Graham's number- where are you?**

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**I think you get the idea. If you have any rational reason to need or desire to go further, then you still have all means available to do so. There is no limiting problem. Instead, the problems become understanding or grasping the vast numbers you create, placing them on a comparative scale with known, comprehensible numbers, not making <u>any</u> errors, having enough time over the span of your entire life to complete the project you start and accomplishing anything at all worthwhile, meaningful or valuable to mathematics in exchange for this tremendous effort.**

---

**No positive value can <u>ever</u> be attained by this manner (or any other) which is even an extremely-small, finite fraction of the value of positive infinity due to the fact that all values which can be generated as such are inescapably finite, regardless of however great.**

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