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**beyond model I (a) & model II (a)**

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two perfect symmetry number theories,  
one preferred

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Actually, there are (at least) two practical, quality methods for performing revised multiplication that comply with perfect symmetry as well as preclude the need for the unit imaginary number, complex number system and every resulting number system (an infinite number).

The main requirement for a perfect symmetry, revised multiplication is:

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$$+a \times +b = +c$$

example-

$$+1 \times +1 = +1$$

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AND

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$$-a \times -b = -c$$

example-

$$-1 \times -1 = -1$$

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With two factors of any possible signs (a & b) ...

- the sign of the product (c) must be determined with all four interactions.

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You might think it is reasonable to question this imperative. I experimented and found it impossible to consistently graph revised multiplication using any scheme where only two interactions (i.e., same signs) were allowed. So, this is a critical directive since anything less leaves any method of revised multiplication incomplete and faulty.

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With two factors of the same sign (+a & +b or -a & -b) ...

- the sign of the product (c) can most logically be determined to be the same as the sign of both factors:

$$+a \times +b = +c$$

OR

$$-a \times -b = -c$$

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Nonetheless, this situation leaves open a few unresolved possibilities or options:

1. The sign of the product (c) is determined jointly by both factors.
2. The sign of the product (c) is determined solely by one factor-

EITHER

a. the sign of the multiplicand (a).

OR

b. the sign of the multiplier (b).

Of course, when the sign of both factors (a & b) is the same, knowing and proving which (if any) is the mathematical reality of the matter is totally ineffectual as well as impossible.

Fortunately, there are two important facts that support this scheme of revised multiplication with two factors of the same sign (+a & +b or -a & -b):

- A. When two positive factors (+a & +b) are involved, the product (c) must be positive to be compliant with measurable, physical results for the sake of applied mathematics.
- B. When two negative factors (-a & -b) are involved, the product (c) must be negative so that it is the symmetrical mirror-image of when two positive factors are involved to prevent the need for the unit imaginary number, etc.

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With two factors of opposite signs (+a & -b or -a & +b) ...

- the sign of the product (c) can be determined in every case but devising exactly how to do so correctly requires careful planning.

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Imperatively, the possibility or option #1 (from page 2) that “the sign of the product (c) is determined jointly by both factors” must be denied or rejected, in this situation, since it would render unachievable the critical directive (from page 2) that “the sign of the product (c) must be determined with all four interactions”.

In other words ...

If it were accepted that the sign of the product (c) is determined jointly by both factors (a & b), then whenever both factors were of opposite signs (+a & -b or -a & +b) the sign of the product (c) could not possibly be determined conclusively.

This leaves only possibility or option #2 (from page 2) that “the sign of the product (c) is determined solely by one factor- either the sign of the multiplicand (a) or the sign of the multiplier (b)”.

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The ramification that any revised multiplication based upon this scheme with two factors of opposite signs (+a & -b or -a & +b) cannot be unconditionally commutative must be understood. If they were allowed to be unconditionally commutative, then the sign of the product (c) would be reversed anytime the order of the two factors was reversed (which would be arbitrarily allowed anytime). Hence, the sign of the product (c) would be changeable and undeterminable.

When using any scheme of revised multiplication with two factors of opposite signs (+a & -b or -a & +b) that is not unconditionally commutative or that is conditionally commutative, I prefer possibility or option #2a (from page 2) for the sign of the product (c) to be determined by the sign of the multiplicand (a).

The only rational reason I can pinpoint for my preference is that I want to know the sign of the product (c) as soon as possible. After all, the multiplicand (a) comes first. I prefer not to have to wait an instant longer until I see the multiplier (b) that comes second.

It is my contention that this single, admittedly-small reason has some real, tangible importance to human education in mathematics. Notwithstanding, schemes of revised multiplication where the sign of the product (c) is determined by the sign of the multiplier (b), instead, are equally valid and definitely, readily constructible by anyone who considers the endeavor sufficiently worthwhile in a likewise manner as I have created two schemes where the sign of the product (c) is determined by the sign of the multiplicand (a).

Overall, schemes of revised multiplication where the sign of the product (c) is determined by the sign of the multiplicand (a) or determined by the sign of the multiplier (b) can be considered inverse forms of one another roughly analogous to equations that can be optionally set to isolate and solve for either of two variables.

In conclusion ...

For one little reason, I have arbitrarily chosen:

- the sign of the product (c) is determined by the sign of the multiplicand (a).

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Consequently ...

- A. When two opposite-signed factors with a positive multiplicand (+a) and a negative multiplier (−b) are involved, the product (c) must be positive since it is determined by the sign of the multiplicand (+a).
- B. When two opposite-signed factors with a negative multiplicand (−a) and a positive multiplier (+b) are involved, the product (c) must be negative since it is determined by the sign of the multiplicand (−a).

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For our purposes, it can now be codified into a reliable, universal rule that the sign of the multiplicand (a) determines the sign of the product (c) in every case.

Please note that only with two factors of the same sign (+a & +b or −a & −b) is revised multiplication unconditionally commutative. Otherwise, conversion between pairs of opposite-signed, identical multipliers (b) is required to obtain two factors of the same sign (+a & +b or −a & −b) where revised multiplication is unconditionally commutative ... if doing so is deemed worthwhile or desirable.

When dealing with two factors of opposite signs (+a & −b or −a & +b), it is the absolute value of the product (c) instead where numerous, arbitrary options suddenly present themselves and become possible. Theoretically, an infinite number of arbitrary, self-consistent models for creating products (c) in the strange non-physical, intangible world where positive and negative real numbers are mixed are equally valid, justifiable and permissible within revised multiplication. This is a fact that Dr. Mark Burgin was the first to ever state in published math papers.

For educational reasons, I strongly recommend seriously considering only extremely-select models that reveals a simple, rational, intuitive algorithm between the absolute values of two, opposite-signed factors (+a & −b or −a & +b) involved. This serves the practical goal well of narrowing the selection to a single best model in order to competently recommend a new, universal standard- esp. since I am aware of only two models (termed “model I-A” & “model II-A”) that meet these quality criteria.

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Fortunately, model I-A and model II-A are inverses of one another and as such, are the only two models possible in accordance with extremely-select design principles. Moreover, they are identical in some respects. The sole foundational contrast (from which all other contrasts are ramifications) between model I-A and model II-A is clearly evident through the function-graph relations via both manifestations- numerically in the functions and visually in the graphs.

Functionally ...

In model I-A, pairs of identical multipliers (b) are numerically opposites.

In model II-A, pairs of identical multipliers (b) are numerically opposites and reciprocals.

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Graphically ...

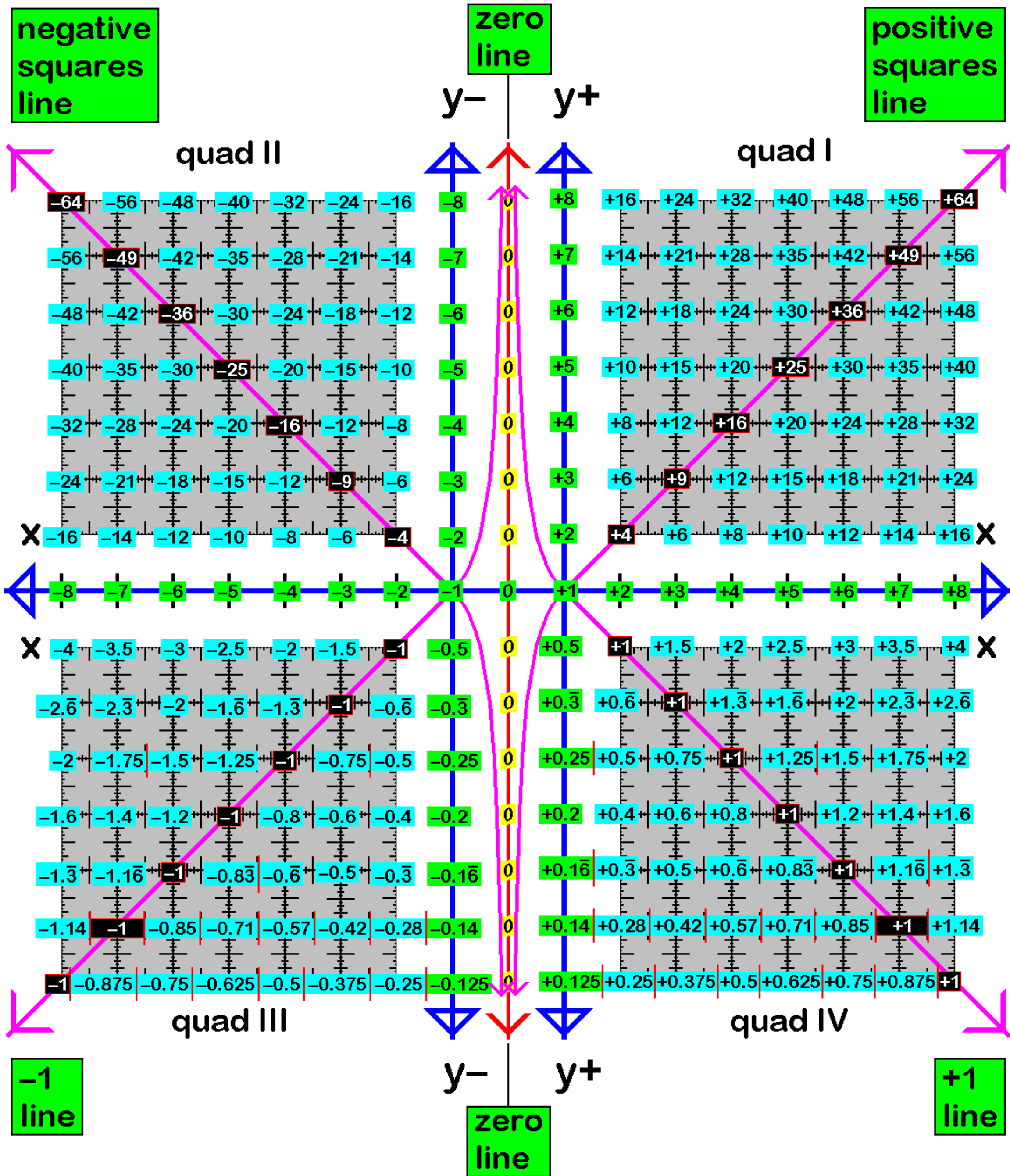
Model I-A and model II-A represent the only two possible relative orientations of the y+ and -y axes within revised multiplication.

revised multiplication- model I-A  
See page 8.

revised multiplication- model II-A  
See page 9.

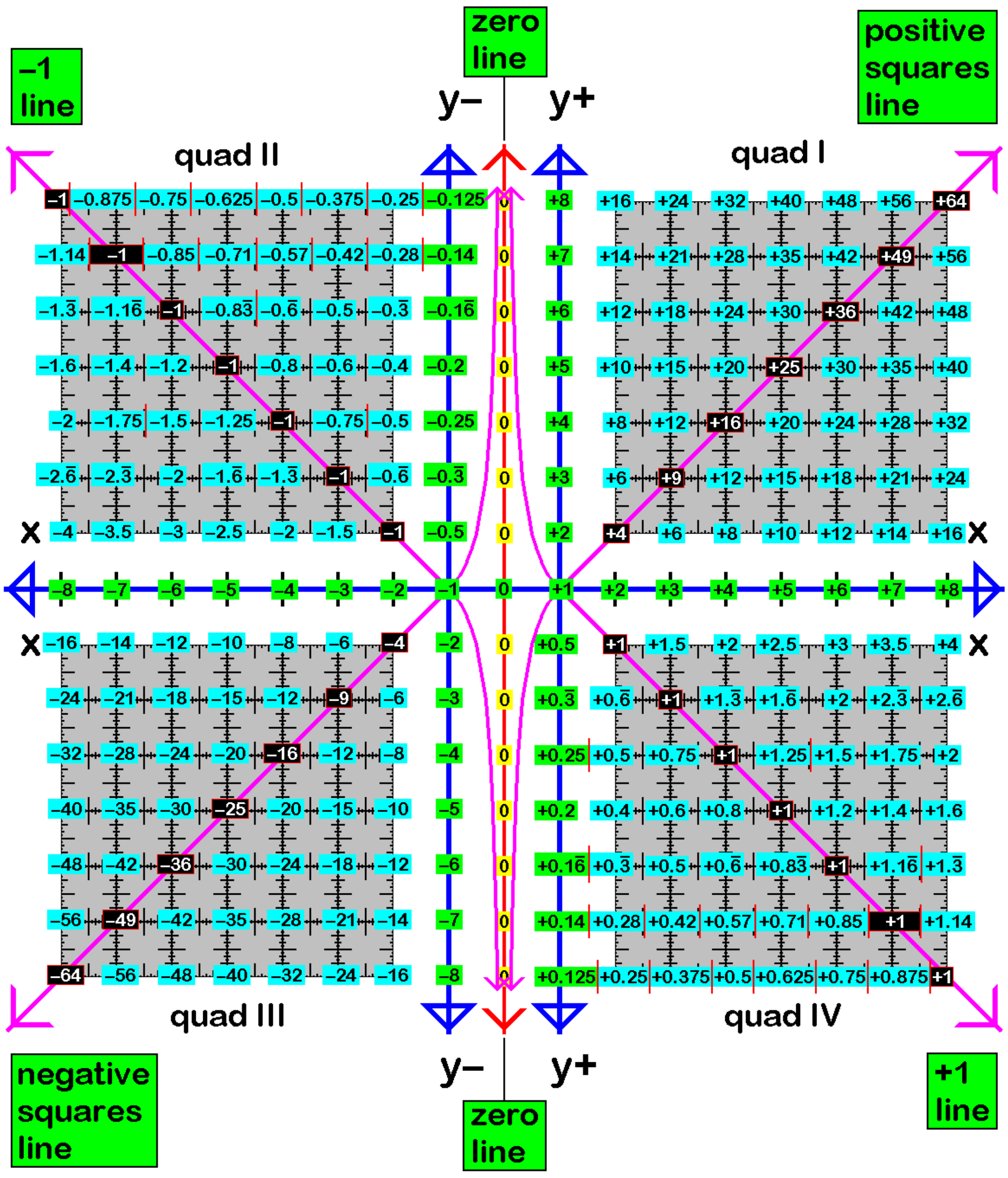
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# revised multiplication





# revised multiplication



**For general interest, comparable examples of revised multiplication under five models follow:**

**model I-A  
(published, written- provided here)**

**model I-B  
(unpublished, unwritten)**

**model II-A  
(unpublished, unwritten)**

**model II-B  
(unpublished, unwritten)**

**the universal standard model  
(published, written).**

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model I-A

conditionally commutative

where the sign of the multiplicand (a)  
determines the sign of the product (c)

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examples-

$$+2 \times +4 = +8$$

$$+2 \times -4 = -8$$

$$+2 \times +0.25 = +0.5$$

$$+2 \times -0.25 = -0.5$$

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$$+2 \times -4 = -8$$

$$+2 \times +4 = +8$$

$$+2 \times -0.25 = -0.5$$

$$+2 \times +0.25 = +0.5$$

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$$-2 \times +4 = -8$$

$$-2 \times -4 = +8$$

$$-2 \times +0.25 = -0.5$$

$$-2 \times -0.25 = +0.5$$

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$$-2 \times -4 = +8$$

$$-2 \times +4 = -8$$

$$-2 \times -0.25 = +0.5$$

$$-2 \times +0.25 = -0.5$$

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model I-B

conditionally commutative

where the sign of the multiplier (b)  
determines the sign of the product (c)

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examples-

$$+2 \times +4 = +8$$

$$-2 \times +4 = +8$$

$$+2 \times +0.25 = +0.5$$

$$-2 \times +0.25 = +0.5$$

---

$$-2 \times +4 = +8$$

$$+2 \times +4 = +8$$

$$-2 \times +0.25 = +0.5$$

$$+2 \times +0.25 = +0.5$$

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$$+2 \times -4 = -8$$

$$-2 \times -4 = -8$$

$$+2 \times -0.25 = -0.5$$

$$-2 \times -0.25 = -0.5$$

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$$-2 \times -4 = -8$$

$$+2 \times -4 = -8$$

$$-2 \times -0.25 = -0.5$$

$$+2 \times -0.25 = -0.5$$

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model II-A

conditionally commutative

where the sign of the multiplicand (a)  
determines the sign of the product (c)

---

examples-

$$+2 \times +4 = +8$$

$$+2 \times -0.25 = +8$$

$$+2 \times +0.25 = +0.5$$

$$+2 \times -4 = +0.5$$

---

$$+2 \times -0.25 = +8$$

$$+2 \times +4 = +8$$

$$+2 \times -4 = +0.5$$

$$+2 \times +0.25 = +0.5$$

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$$-2 \times +0.25 = -8$$

$$-2 \times -4 = -8$$

$$-2 \times +4 = -0.5$$

$$-2 \times -0.25 = -0.5$$

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$$-2 \times -4 = -8$$

$$-2 \times +0.25 = -8$$

$$-2 \times -0.25 = -0.5$$

$$-2 \times +4 = -0.5$$

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**model II-B**

**conditionally commutative**

**where the sign of the multiplier (b)  
determines the sign of the product (c)**

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**examples-**

$$\begin{aligned} +2 \times +4 &= +8 \\ -0.5 \times +4 &= +8 \end{aligned}$$

$$\begin{aligned} +2 \times +0.25 &= +0.5 \\ -0.5 \times +0.25 &= +0.5 \end{aligned}$$

---

$$\begin{aligned} -0.5 \times +4 &= +8 \\ +2 \times +4 &= +8 \end{aligned}$$

$$\begin{aligned} -0.5 \times +0.25 &= +0.5 \\ +2 \times +0.25 &= +0.5 \end{aligned}$$

---

$$\begin{aligned} +0.5 \times -4 &= -8 \\ -2 \times -4 &= -8 \end{aligned}$$

$$\begin{aligned} +0.5 \times -0.25 &= -0.5 \\ -2 \times -0.25 &= -0.5 \end{aligned}$$

---

$$\begin{aligned} -2 \times -4 &= -8 \\ +0.5 \times -4 &= -8 \end{aligned}$$

$$\begin{aligned} -2 \times -0.25 &= -0.5 \\ +0.5 \times -0.25 &= -0.5 \end{aligned}$$

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**universal standard model**

**unconditionally commutative**

**where the arbitrary “rule of signs”  
determines the sign of the product (c)**

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**examples-**

$$+2 \times +4 = +8$$

$$-2 \times -4 = +8$$

$$+2 \times +0.25 = +0.5$$

$$-2 \times -0.25 = +0.5$$

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$$-2 \times -4 = +8$$

$$+2 \times +4 = +8$$

$$-2 \times -0.25 = +0.5$$

$$+2 \times +0.25 = +0.5$$

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$$+2 \times -4 = -8$$

$$-2 \times +4 = -8$$

$$+2 \times -0.25 = -0.5$$

$$-2 \times +0.25 = -0.5$$

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$$-2 \times +4 = -8$$

$$+2 \times -4 = -8$$

$$-2 \times +0.25 = -0.5$$

$$+2 \times -0.25 = -0.5$$

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## **comparison- A & B models**

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**Through examples of revised multiplication, a pattern should now become evident within the computational comparisons between model I-A & model I-B and model II-A & model II-B that renders the only practical distinction trivial.**

**When dealing with inverse forms of revised multiplication within which the sign of the product (c) is determined either by the sign of the multiplicand (a) or by the sign of the multiplier (b), the products (c) yielded in revised multiplication are comparatively-identical in every case if the order of the two factors are reversed. Therefore, technically elaborating two more schemes of revised multiplication where the sign of the product (c) is determined by the sign of the multiplier (b) would be an unproductive effort that reveals nothing new since this has already been accomplished for two equivalent schemes of revised multiplication where the sign of the product (c) is determined by the sign of the multiplicand (a).**

**By this logic, choosing to work only with forms of revised multiplication where the sign of the multiplicand (a) determines the sign of the product (c), instead of the sign of the multiplier (b), can be regarded likewise to choosing any other convention or standard for reasons of convenience and familiarity.**

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## comparison- I & II models

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Upon detailed examination, I prefer model I-A over model II-A for seven reasons:

1. Model I-A has simpler algorithms for revised multiplication to understand and use than model II-A. This is a very important advantage from the practical standpoint of human education.
2. Model I-A requires only two unique algorithms to handle all four possible interactions of signed factors (a & b) in revised multiplication whereas model II-A requires four unique algorithms.
3. Model I-A has two unique algorithms that are perfectly symmetrical mirror-images of one another. This means the two unique algorithms are extremely similar to one another numerically having identical absolute values in every case yet comparatively-opposite signs for the products (+c or -c) as their sole distinction.
4. Model I-A has two unique algorithms, exclusively positive and exclusively negative, that can be correctly classified as positive and negative applications, respectively, of a single, sign-less core algorithm. Consequently, I am certain model I-A is the simplest, complete, self-consistent model of revised multiplication possible theoretically that is suitable as a new, universal standard.
5. Model I-A has identical multipliers that are opposites which are much easier and quicker to calculate without significant risk of errors than for model II-A where identical multipliers are opposites AND reciprocals. Furthermore, model I-A seems to require only a “blindness” to the sign of the multiplier (b) to get the correct answer using revised multiplication where the sign of the product (c) is determined solely by the sign of the multiplicand (a) in every case.
6. Model I-A is similar to the universal standard model already in use (i.e., conventional multiplication) in more ways than model II-A. Model I-A is identical to the universal standard model in two out of four possible interactions of signed factors (a & b) whereas model II-A is identical in only one out of four. Moreover, model I-A has absolute values of products (c) that are comparatively-identical in every case to the universal standard model. The only practical distinction is the signs of the products (c) are comparatively-opposite in two out of four possible interactions of signed factors (a & b).

Specifically ...

- A. When two negative factors (-a & -b) are involved, the product (c) in model I-A is negative instead of positive as in the universal standard model.
- B. When two opposite-signed factors with a positive multiplicand (+a) and a negative multiplier (-b) are involved, the product (c) in model I-A is positive instead of negative as in the universal standard model.

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7. Model I-A just happens to be the least disruptive change theoretically possible (out of ALL of the perfect symmetry models) from the present, universal standard model that humans just happen to use in the early 21<sup>st</sup> century. All you have to do is change the “rule of signs” in half (i.e., two out of four) of the cases. Even though the universal standard model is poorly designed to such an extreme that its detailed characteristics are ultimately unimportant theoretically, this is a very lucky coincidence as well as an ideal opportunity for improvement that should not be delayed or disregarded.

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Conversely, I cannot think of any reasons to prefer model II-A.

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**eight extended real number continuums**

**(eight revised slope systems)**

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**Note: For additional details, see the section, “the extended real number continuum”, within paper I.**

**perfect symmetry number theory  
model I**

**<http://www.symmetryperfect.com/papers/paper-1.pdf>**

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**Within the perfect symmetry number theory, one out of eight possible models of the extended real number continuum had to be arbitrarily chosen. In turn, the chosen model is one of two foundations of revised analytic geometry and revised calculus. The other foundation is the analytic/numerical ramification of revised arithmetic.**

**In no sense was the one model chosen any better or worse than any of the seven models not chosen. In fact, all eight models are as select as possible. They share in common that every unique extended real number (and revised slope of every ray) is represented exactly once on a 360° circle and every pair of geometrically-opposite rays appropriately have numerically-opposite revised slopes. Consequently, an exact geometric relation of every value to one another throughout the entire graph has to be maintained. However, there are eight ways to represent this exact relation with positive and negative values distributed across the four quadrants.**

**My choice was determined simply by adopting the familiar convention where the highest positive real numbers are in quadrant I with absolute values that increase in the counter-clockwise direction or decrease in the clockwise direction.**

**For all of the example graphs, the circular depiction was arbitrarily used although the congruent, linear depiction could have been used as well, of course.**

**all eight extended real number continuums  
circular depiction**

**<http://www.symmetryperfect.com/R-cont/8-cont.pdf>**

**There is an “A” & “B” series which are inverses of one another.**

In the “A” series, the absolute values of their positive and negative real numbers generally increase in the counter-clockwise direction or decrease in the clockwise direction (except when they hit zero).

[the chosen model]

positive quads I & IV  
negative quads II & III

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-a-p-1-4-n-2-3.pdf>

positive quads I & II  
negative quads III & IV

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-a-p-1-2-n-3-4.pdf>

positive quads II & III  
negative quads I & IV

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-a-p-2-3-n-1-4.pdf>

positive quads III & IV  
negative quads I & II

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-a-p-3-4-n-1-2.pdf>

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In the “B” series, the absolute values of their positive and negative real numbers generally increase in the clockwise direction or decrease in the counter-clockwise direction (except when they hit zero).

positive quads I & IV  
negative quads II & III

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-b-p-1-4-n-2-3.pdf>

positive quads I & II  
negative quads III & IV

<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-b-p-1-2-n-3-4.pdf>

positive quads II & III  
negative quads I & IV

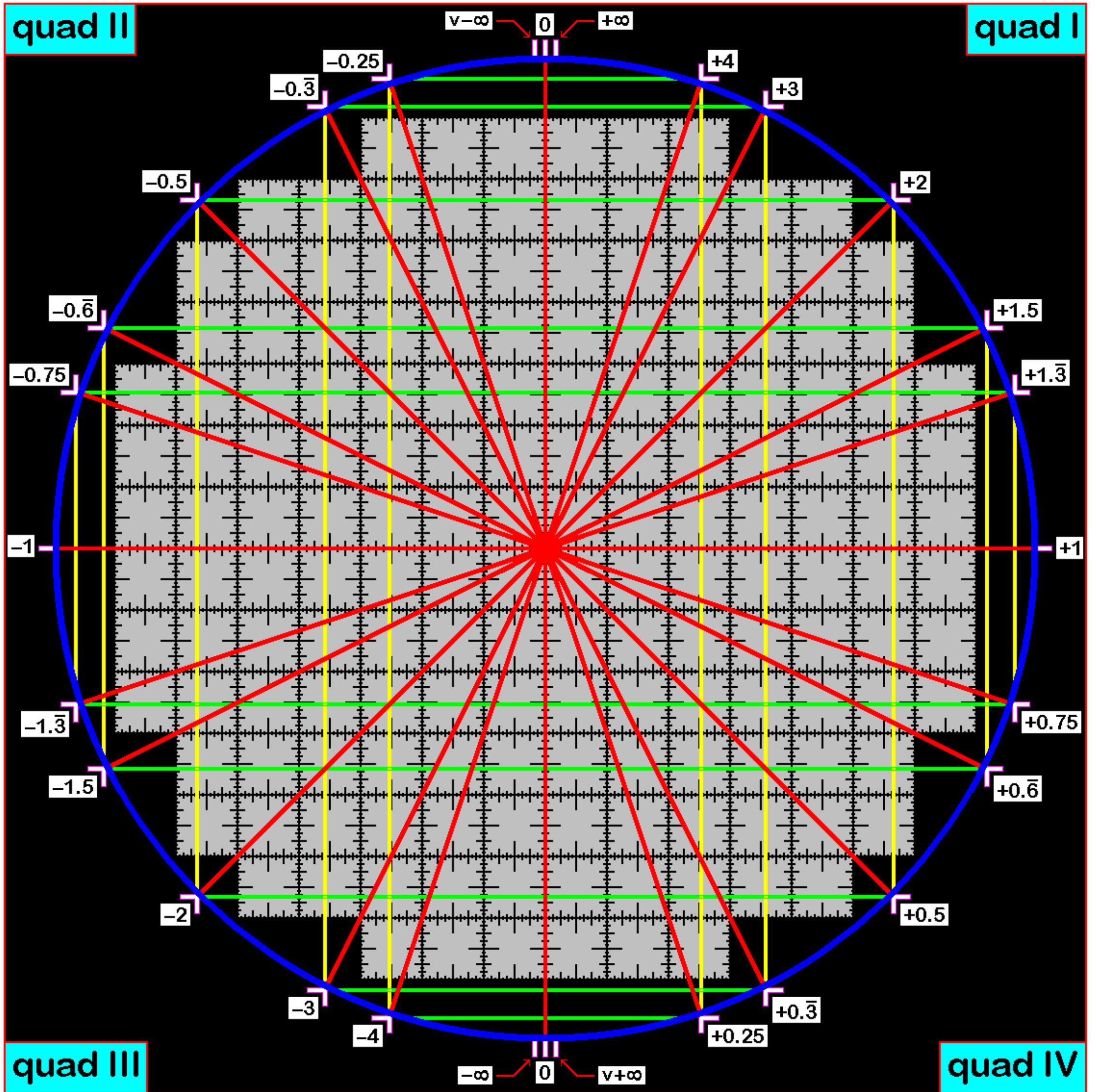
<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-b-p-2-3-n-1-4.pdf>

positive quads III & IV  
negative quads I & II

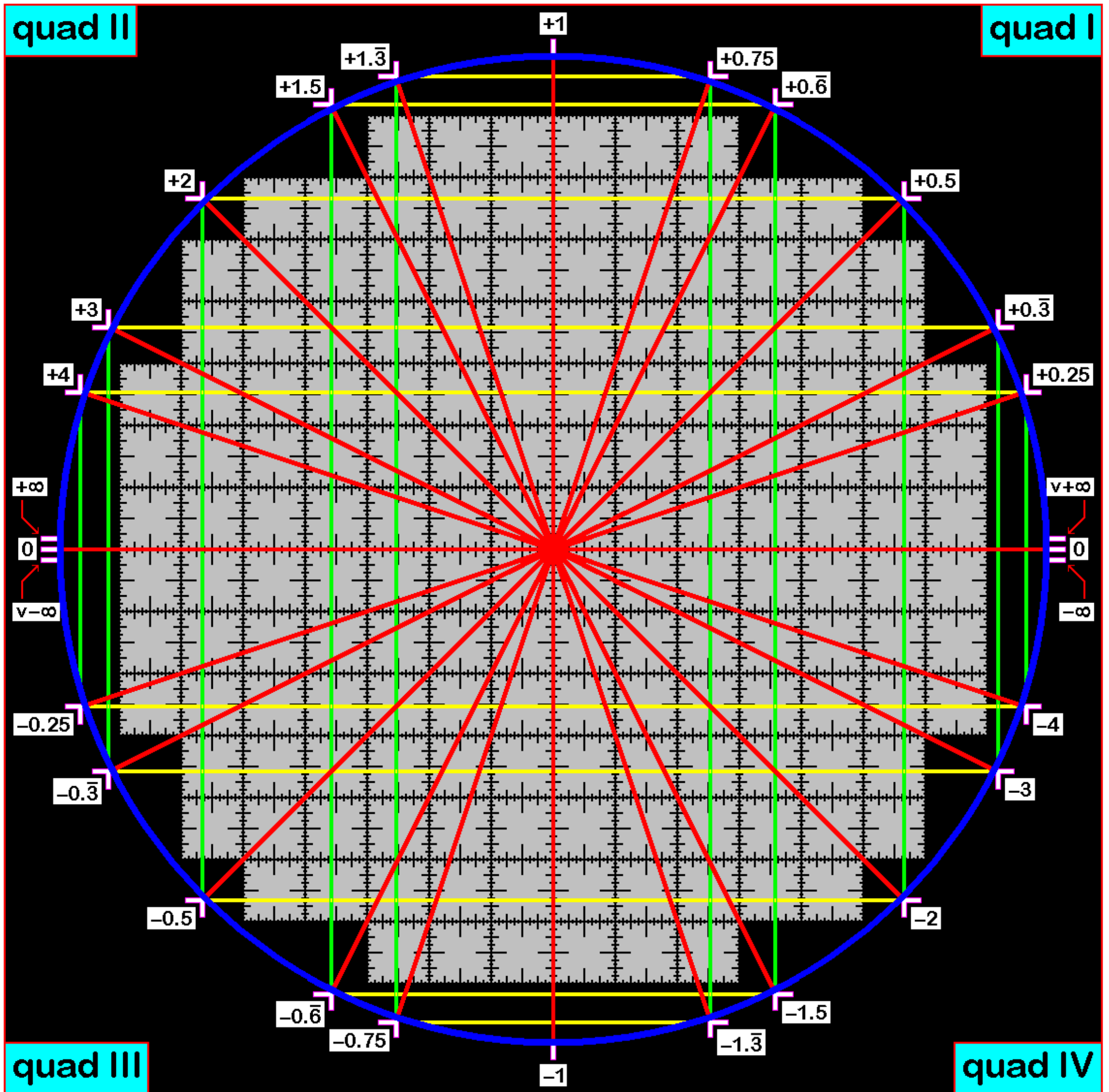
<http://www.symmetryperfect.com/R-cont/graphs/cont-circ-b-p-3-4-n-1-2.pdf>

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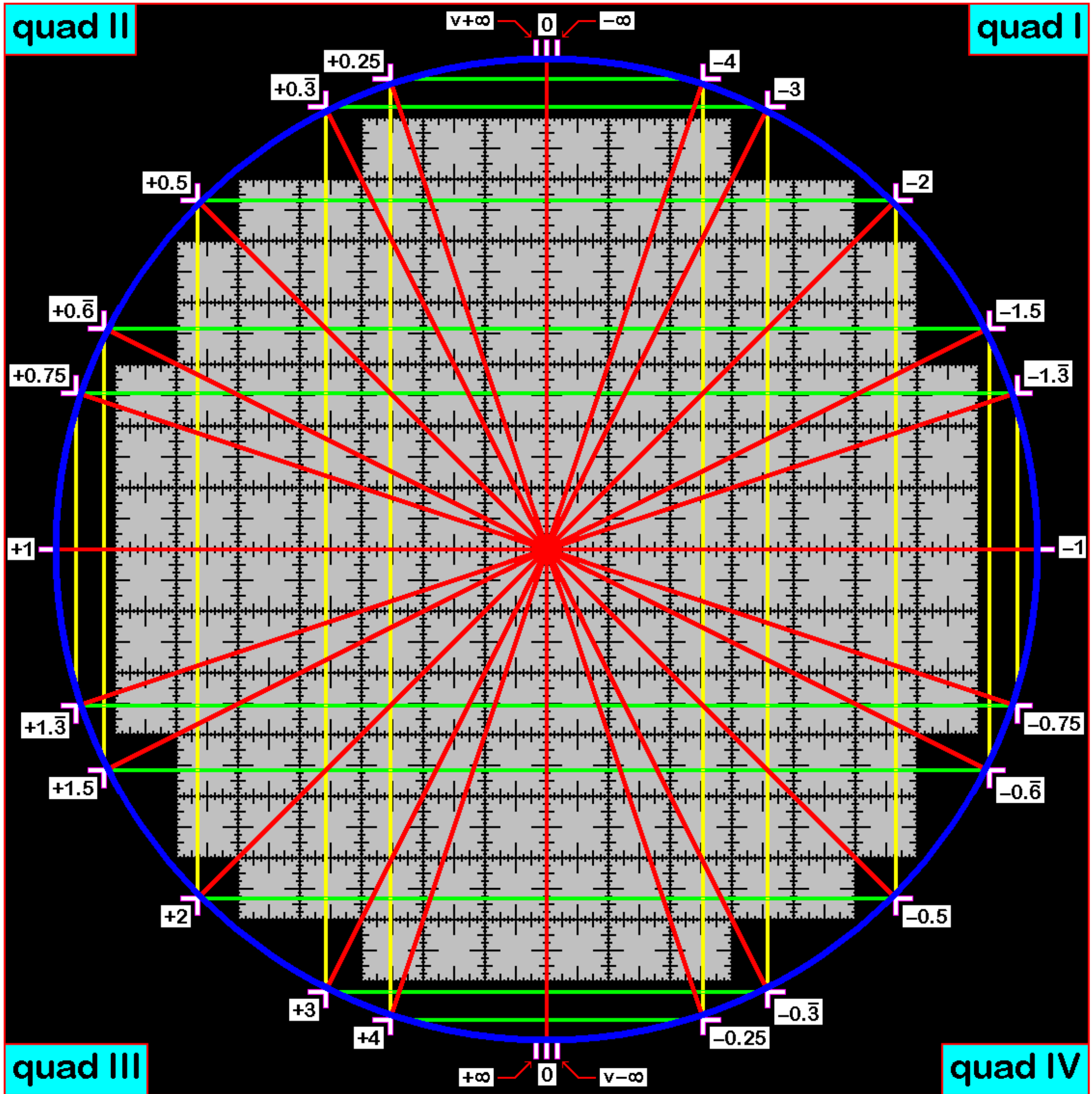
the extended real number continuum



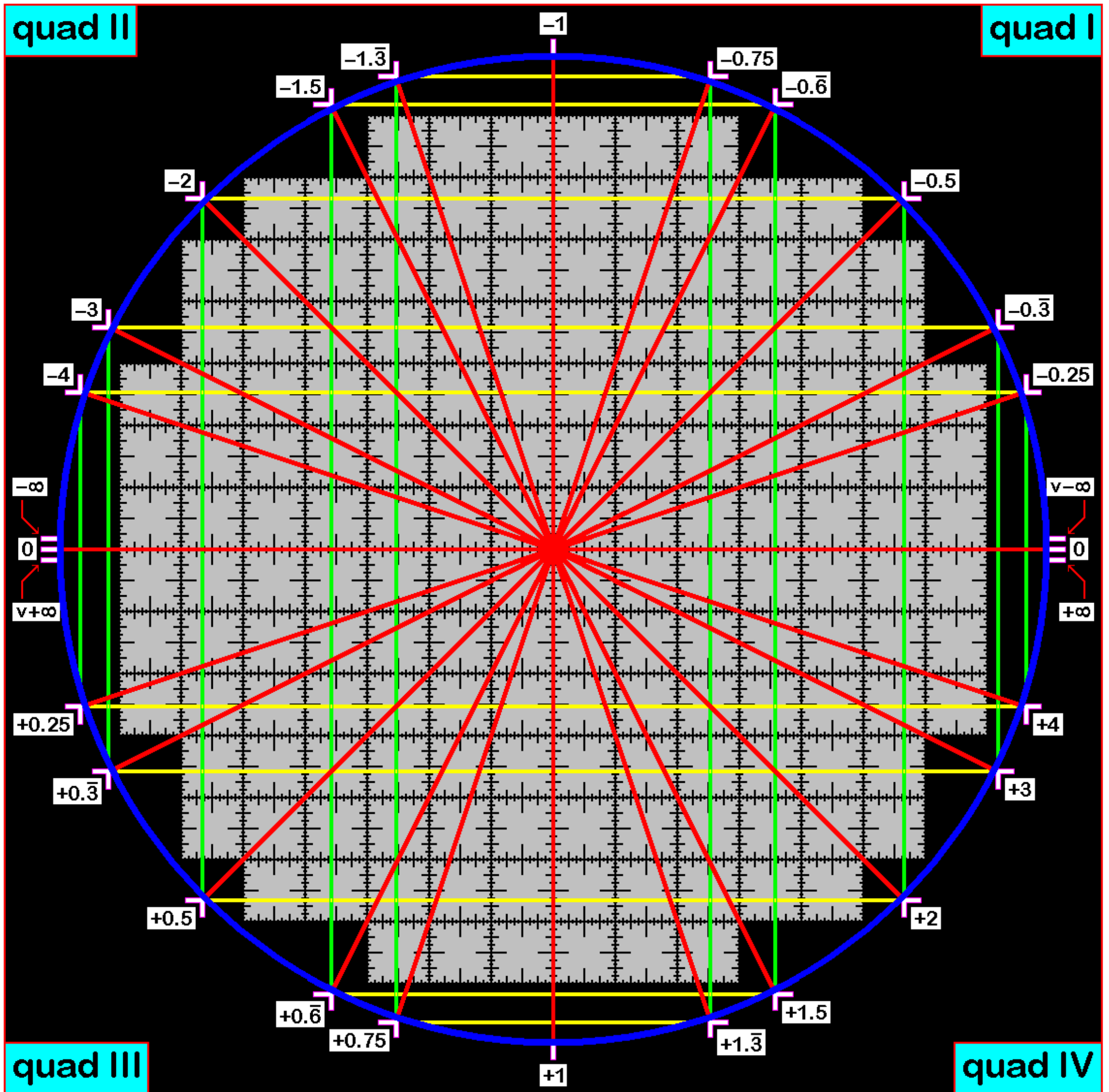
the extended real number continuum



the extended real number continuum

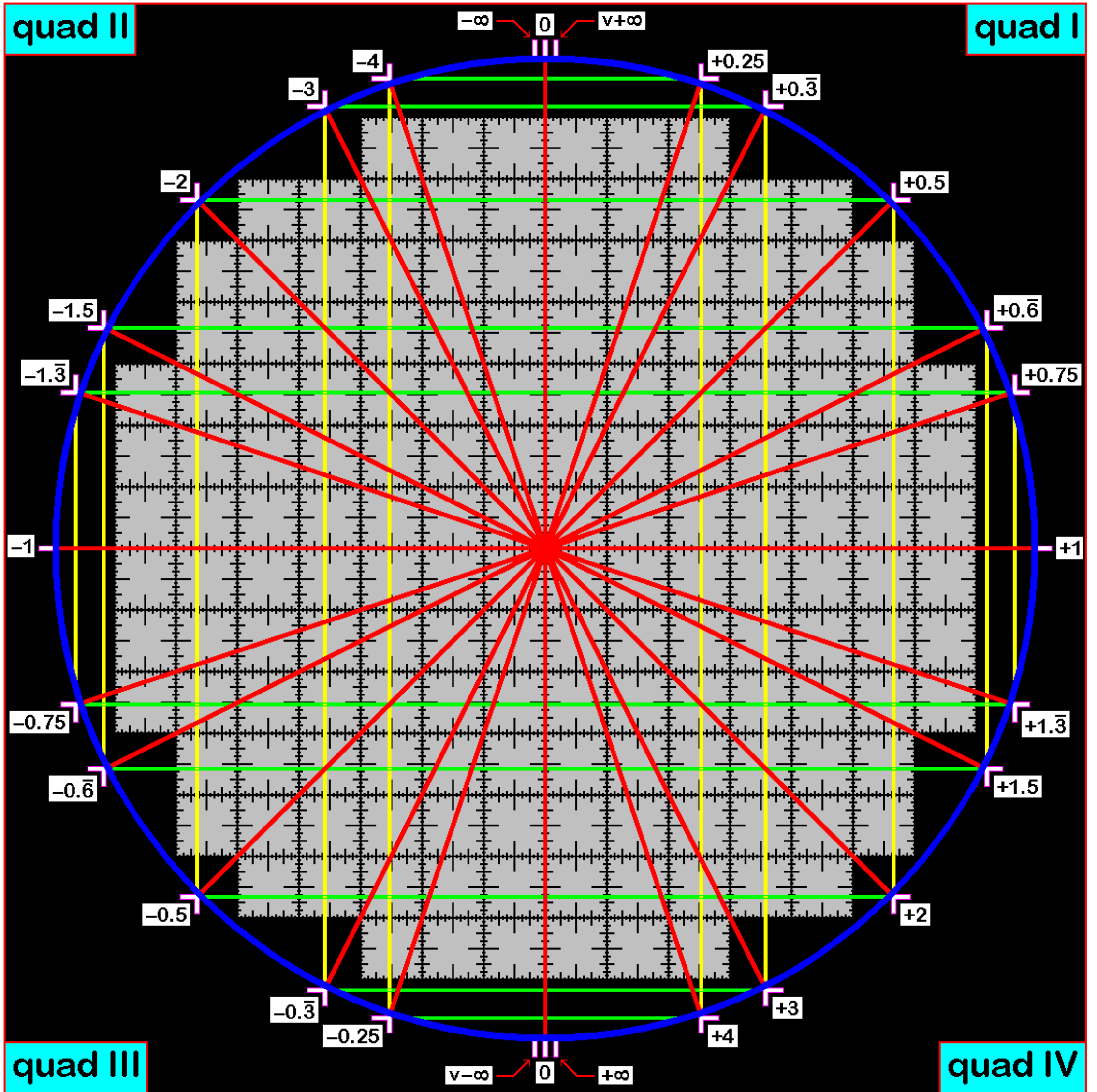


the extended real number continuum

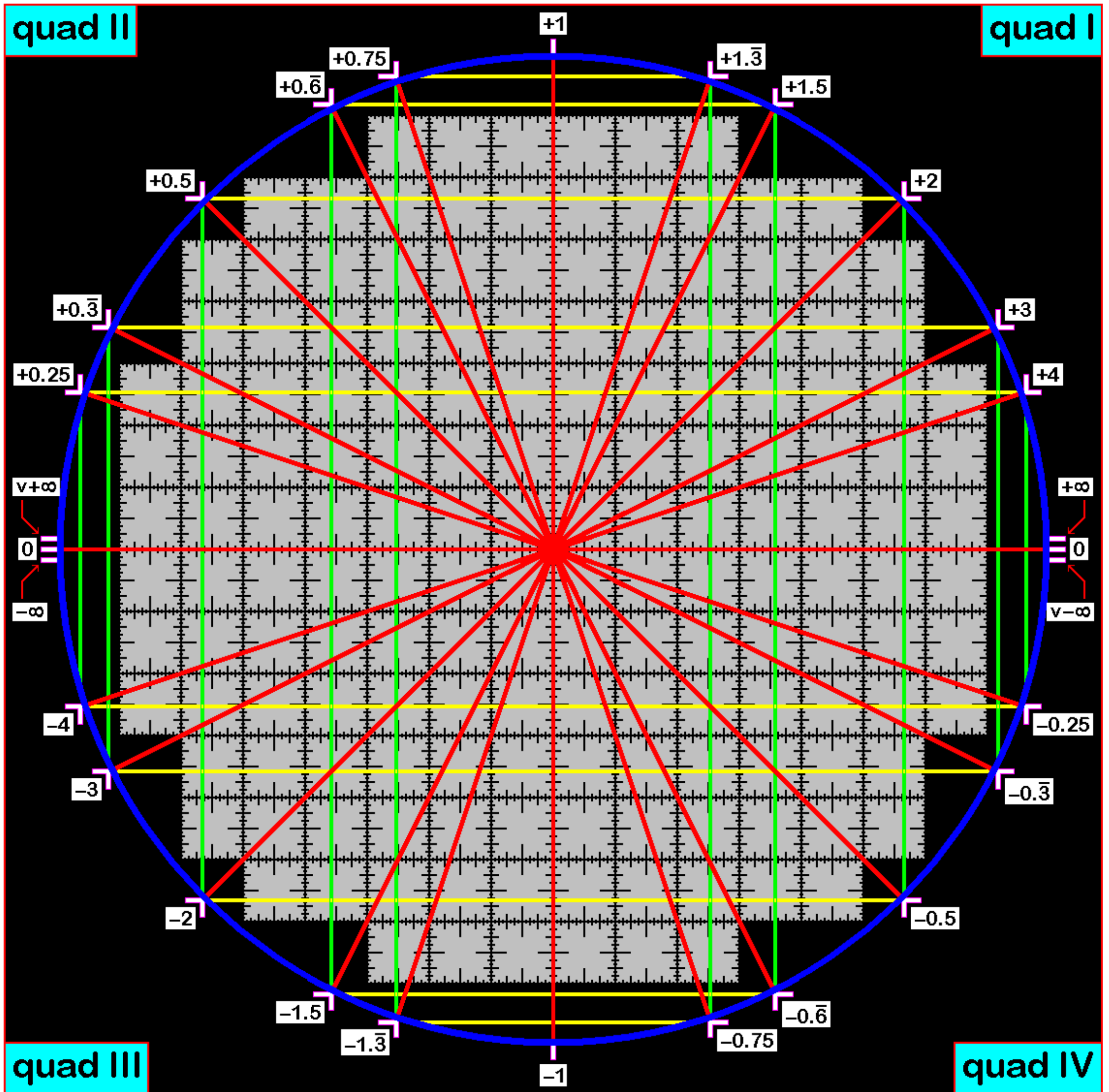




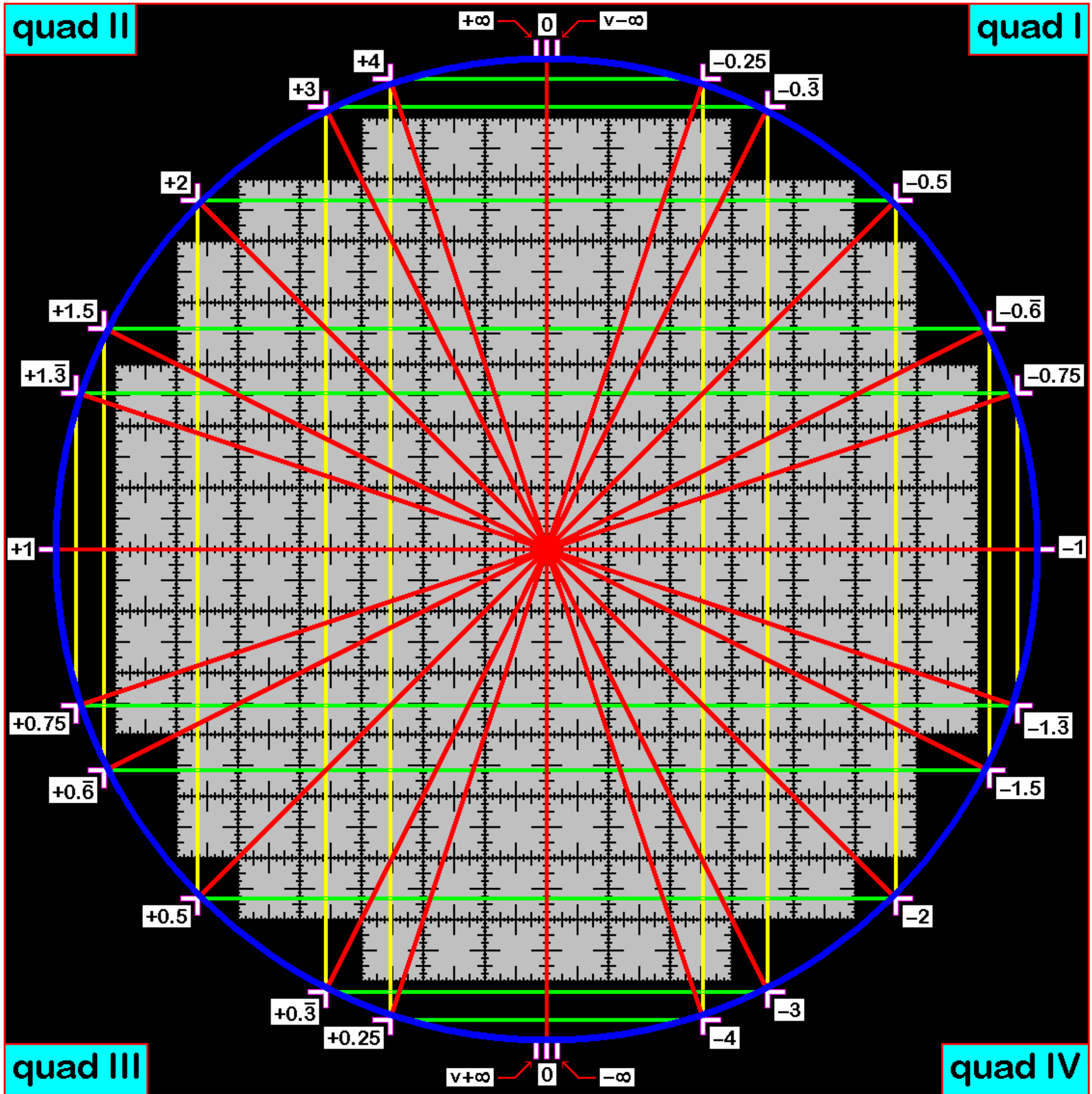
the extended real number continuum



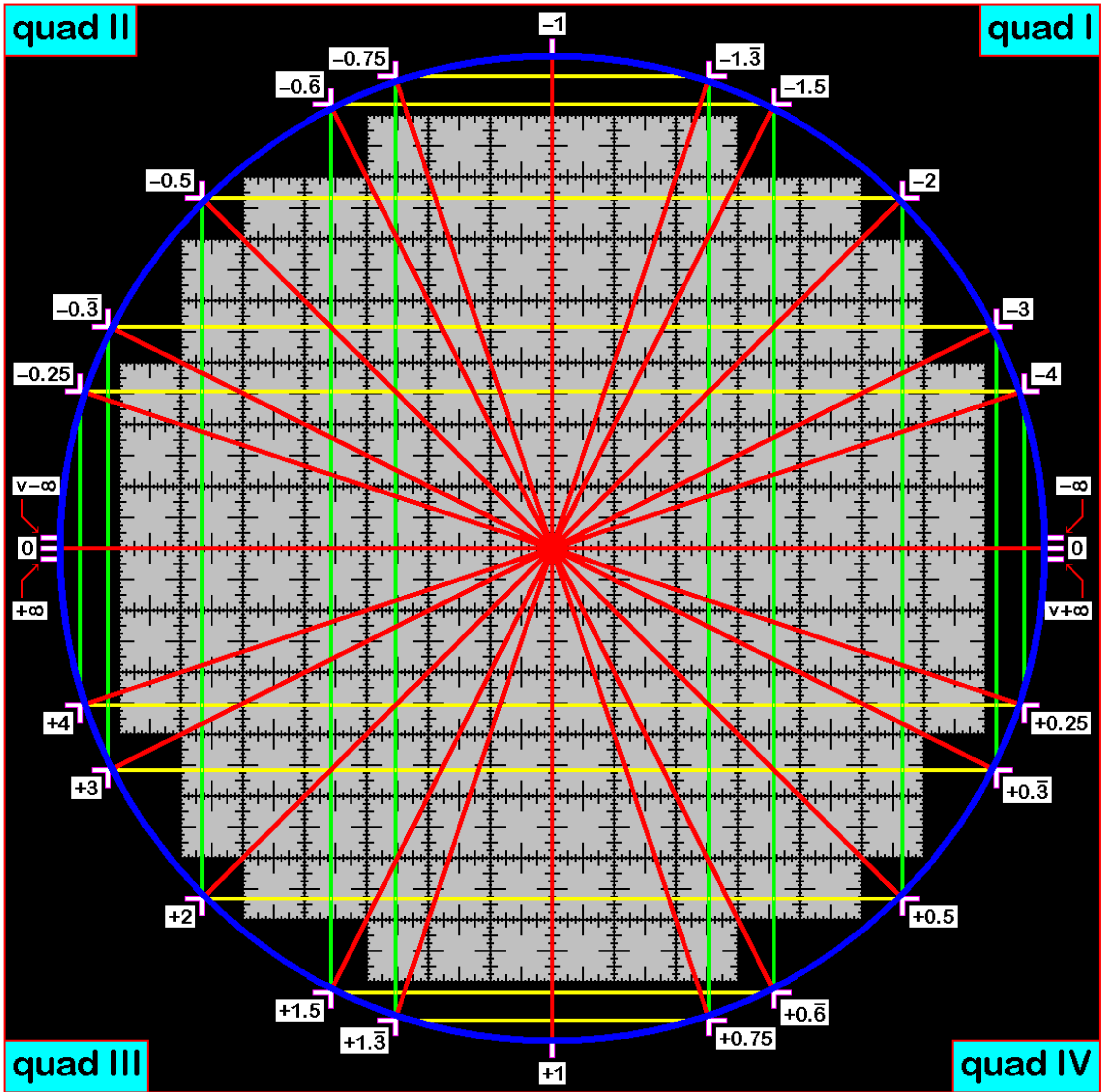
the extended real number continuum



the extended real number continuum



the extended real number continuum



## **conclusions**

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**The 2 possible perfect symmetry number theories taken in tandem with the 8 possible extended real number continuums (and revised slope systems) ramify that a total of 16 complete systems of revised mathematics including arithmetic, algebra, analytic geometry, analytic trigonometry and calculus are constructible based upon perfect symmetry and other quality design principles.**

**Hypothetically (to the extreme), this means that if an advanced alien civilization that understood mathematics extremely well and practiced it correctly were to communicate with or visit Earth, they would be equally likely (more or less) to use any of 16 mathematical systems of highest quality yet they would almost certainly not use an inferior mathematical system identical or similar to that used by the human race.**

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